DISCOVERY

Mean displacement law for black body radiations and temperature of electromagnetic waves

Ganesa Moorthy C[1*], Udhaya Sankar G[2]

To Cite:
Moorthy CG, Sankar GU. Mean displacement law for black body radiations and temperature of electromagnetic waves. *Discovery* 2023; 59: e85d1279

Author Affiliation:
[1]Department of Mathematics, Alagappa University, Karaikudi 630003, India
[2]Department of Physics, Alagappa University, Karaikudi 630003, India

[*]**Corresponding author**
Department of Mathematics, Alagappa University, Karaikudi 630003,
India
Email: ganesamoorthyc@gmail.com

Peer-Review History
Received: 30 April 2023
Reviewed & Revised: 04/May/2023 to 05/June/2023
Accepted: 09 June 2023
Published: July 2023

Peer-Review Model
External peer-review was done through double-blind method.

Discovery
pISSN 2278–5469; eISSN 2278–5450

DISCOVERY SCIENTIFIC SOCIETY

ABSTRACT

Wien's displacement law was derived from mode value of the very old Planck's distribution for black body radiations. One more new displacement law is derived from mean value of the Planck's distribution, by evaluating two integrals. They are used to propose definitions for temperature of electromagnetic waves.

Keywords: Planck's distribution, Wien's displacement law, Mean of a distribution.

1. INTRODUCTION

The historical Planck's distribution was established mathematically (Planck, 1901). This distribution provides intensities of radiated rays from a black body corresponding to different wavelengths. A black body is an ideal body that can emit gamma rays, X-rays, visible light rays, non-visible light rays and heat rays and that can absorb all these rays without any constraint. Let us consider the very old *Planck's distribution* (Planck, 1901) for intensity of radiated waves with wavelength λ from a black body with temperature T in the form

$$u(\lambda) = \frac{8\pi hc}{\lambda^5}\left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1}.$$

Let us fix the following constants which are expressed only in terms of metre, kilogram, second and Kelvin.

Planck's constant for light waves $h = 6.626 \times 10^{-34}$.

Planck's constant for radio waves ⓗ $= 17.57889908 \times 10^{-20}$.

Boltzmann constant $= 1.38 \times 10^{-23}$.

Speed of light in vacuum $c = 2.9979 \times 10^{8}$.

Wien universal constant $b = 2.8978 \times 10^{-3}$.

Planck's distribution was already studied in Moorthy, (2019) and Moorthy and Sankar, (2023) for different purposes by using same unit expressions. In Moorthy and Sankar, (2023), the factor 10^{-20} is missing in Planck's constant for radio waves which led to some miscalculations. However, Moorthy and Sankar, (2023) will provide a common reference for all required references for technical terms and theory to be discussed in the present article.

If the equation $\frac{d\,u(\lambda)}{d\,\lambda} = 0$ is solved for $\lambda = \lambda_{max}$, then the following relation is obtained:

$$\lambda_{max} T = b.$$

This relation is called *Wien's displacement law*. It is possible to verify that $\frac{d^2u(\lambda)}{d\lambda^2} < 0$ for the value $\lambda = \lambda_{max}$. Thus, $\lambda = \lambda_{max}$ gives the unique maximum value for the Planck's distribution $u(\lambda)$. In terms of mathematical statistics, it is possible to say that $\lambda = \lambda_{max}$ gives the unique mode value for the Planck's distribution $u(\lambda)$. Let us find the mean value $\lambda = \lambda_{avg}$ of the Planck's distribution satisfying a corresponding displacement law in the form $\lambda_{avg}T = a\ constant$. These laws will be used to propose definitions for temperature of electromagnetic waves.

Note that $\frac{1}{\int_0^\infty u(\lambda)\, d\lambda} u(\lambda)$ provides a probability density function in the sense that the integral of this non negative density function from zero to infinity with respect to the variable λ gives the value one. This probability density function can be used to find expected value or mean value of the corresponding probability distribution. This particular mean value is called the mean value of the Planck's distribution. Thus, the mean of the Planck's distribution is $\left(\int_0^\infty \lambda\, u(\lambda) d\lambda\right) / \left(\int_0^\infty u(\lambda)\, d\lambda\right)$.

2. MEAN DISPLACEMENT LAW

Let us recall that the mean of the Planck's distribution is the ratio $\left(\int_0^\infty \lambda\, u(\lambda) d\lambda\right) / \left(\int_0^\infty u(\lambda)\, d\lambda\right)$. To evaluate this ratio, the following integral formula is required for the cases $s = 3$ and $s = 4$: $\int_0^\infty \frac{x^{s-1}}{e^x - 1}\, dx = \zeta(s)\Gamma(s)$; which are given in terms of the standard Riemann zeta function $\zeta(s)$ and in terms of the standard gamma function $\Gamma(s)$. So, let us use the known values $\zeta(3) = 1.2020569, \zeta(4) = 1.0823232, \Gamma(3) = 2! = 2, \Gamma(4) = 3! = 6$. To evaluate numerator integral and denominator integral, let us consider a common substitution $x = \frac{hc}{\lambda kT}$ or $\lambda = \frac{hc}{xkT}$ so that $dx = -\frac{hc}{kT\lambda^2}\, d\lambda$ or $d\lambda = -\frac{kT\lambda^2}{hc}\, dx$. This common substitution leads to integrals containing factor integrals $\int_0^\infty \frac{x^{s-1}}{e^x - 1}\, dx = \zeta(s)\Gamma(s)$, for the cases $s = 3$ and $s = 4$.

Then the numerator integral has the value $8\pi \times \frac{(kT)^3}{(hc)^2} \times 1.2020569 \times 2$ and the denominator integral has the value $8\pi \times \frac{(kT)^4}{(hc)^3} \times 1.0823232 \times 6$. Then the value of the ratio $\left(\int_0^\infty \lambda\, u(\lambda) d\lambda\right) / \left(\int_0^\infty u(\lambda)\, d\lambda\right)$ becomes $5.3288843 \times 10^{-3} \times \frac{1}{T}$. Thus, whenever T is fixed, an average value for wavelength for all rays emitted from a black body with temperature T is $\lambda_{avg} = 5.3288843 \times 10^{-3} \times \frac{1}{T}$. So, the following new *mean displacement law* is obtained:

$\lambda_{avg}T = 5.3288843 \times 10^{-3}$.

The unit of the constant given in the right-hand side is "metre Kelvin". This proposes a definition for temperature of the radiated wave with wavelength λ_{avg} as $T = 5.3288843 \times 10^{-3} \times \frac{1}{\lambda_{avg}}$. This proposes a definition for temperature of a radiated wave with wavelength λ as $T = 5.3288843 \times 10^{-3} \times \frac{1}{\lambda}$. This one provides a definition for temperature for electromagnetic waves.

Temperature of electromagnetic waves through mean displacement law

Details and references may be found in (Moorthy and Sankar, 2023) for the followings. An energy wave is defined as an electromagnetic wave, if it can travel through vacuum with speed c, the speed of light in vacuum. There was no precise definition for electromagnetic waves and there were confusions over identifying electromagnetic waves by different individual persons. However, the present article will follow the definition mentioned above. An electromagnetic wave contains only electric field component wave when its wavelength is less than $1.063022744 \times 10^{-3}$ m, approximately and in this case, it is called an electric field wave.

An electromagnetic wave contains only magnetic field component wave when its wavelength is greater than $1.063022744 \times 10^{-3}$ m, approximately and in this case, it is called a magnetic field wave. All light rays, X-rays, gamma rays and the rays radiated from a black body are electric field waves. All radio waves are magnetic field waves and all gravitational waves are magnetic field waves. These facts do not contradict Maxwell equations for electromagnetic waves. Alpha rays, beta rays and electron rays produced by magnetrons are not electromagnetic waves according to the definition mentioned above, because they do not have speed equal to c.

Let us now define temperature of an electric field wave more directly from the definition of temperature of radiated waves, because all electric field waves may be considered as radiated waves. Let us consider an electric field wave with wavelength λ. Let T_λ denote the temperature of this electric field wave to define this concept. Then $T_\lambda = 5.3288843 \times 10^{-3} \times \frac{1}{\lambda}$. This one provides a formula (and definition) for temperature T_λ of an electromagnetic wave with wavelength λ, when it is an electric field wave. For example, a gamma ray with wavelength 10^{-12} m has temperature 5.3288843×10^9 Kelvin. An X-ray with wavelength 10^{-10} m has temperature 5.3288843×10^7 Kelvin. An infrared ray with wavelength 10^{-3} m has temperature 5.3288843 Kelvin.

Let us recall that $E = h\nu$ is the Planck's equation for electric field waves and $E = ⓗ\lambda$ is the Planck's equation for magnetic field waves. Here ν refers to frequency and λ refers to wavelength. To define temperature of a magnetic field wave, let us begin with an electric field wave with wavelength λ_e and a magnetic field wave with wavelength λ_m. Suppose that these two waves have same energy E so that λ_e depends on λ_m and λ_m depends on λ_e. The dependence can be obtained from the relation

$$h\frac{c}{\lambda_e} = E = ⓗ\lambda_m$$

$$\lambda_m = \frac{hc}{ⓗ\lambda_e}$$

$$\lambda_e = \frac{hc}{ⓗ\lambda_m}$$

$$\lambda_e = \frac{1.129996 \times 10^{-6}}{\lambda_m}.$$

For a given magnetic field wave with wavelength λ_m, the relation $\lambda_e = \frac{1.129996\times10^{-6}}{\lambda_m}$ can be applied to find the wavelength λ_e of an electric field wave such that both waves have equal energy so that both waves have same temperature. Thus, if T_{λ_e} and T_{λ_m} represent temperature of the electric field wave and of the corresponding magnetic field wave with equal energy, then $T_{\lambda_m} = T_{\lambda_e} = 5.3288843 \times 10^{-3} \times \frac{1}{\lambda_e}$, when $\lambda_e = \frac{1.129996\times10^{-6}}{\lambda_m}$. More explicitly, $T_{\lambda_m} = 4.7158435 \times 10^3 \times \lambda_m$. Thus, the formula for temperature of a magnetic field wave with wavelength λ is $T_\lambda = 4.7158435 \times 10^3 \times \lambda$. There are two formulas for temperature of an electromagnetic wave obtained through mean displacement law; one for an electric field wave and another one for a magnetic field wave.

Temperature of electromagnetic waves through Wien's displacement law

Let us now propose definition of temperature of electromagnetic waves in terms of Wien's displacement law. Wien's displacement law for electric field waves is the following: $\lambda_{max}T = 2.8978 \times 10^{-3}$, in which the unit for the constant given in the right-hand side is "metre Kelvin". To define temperature of an electric field wave through Wien's displacement law, let us consider an electric field wave with wavelength λ. Let T_λ denote the temperature of this electric field wave to define this concept. Then $T_\lambda = 2.8978 \times 10^{-3} \times \frac{1}{\lambda}$. This one provides a formula (and definition) for temperature T_λ of an electromagnetic wave with wavelength λ, when it is an electric field wave.

For example, a gamma ray with wavelength 10^{-12} m has temperature 2.8978×10^9 Kelvin. An X-ray with wavelength 10^{-10} m has temperature 2.8978×10^7 Kelvin. An infrared ray with wavelength 10^{-3} m has temperature 2.8978 Kelvin. To define temperature of a magnetic field wave through the Wien's displacement law, let us begin with an electric field wave with wavelength λ_e and a magnetic field wave with wavelength λ_m. Suppose that these two waves have same energy E so that λ_e depends on λ_m and λ_m depends on λ_e. The dependence can be obtained from the relation $\lambda_e = \frac{1.129996\times10^{-6}}{\lambda_m}$.

For a given magnetic field wave with wavelength λ_m, the relation $\lambda_e = \frac{1.129996\times10^{-6}}{\lambda_m}$ can be applied to find the wavelength λ_e of an electric field wave such that both waves have equal energy so that both waves have same temperature. Thus, if T_{λ_e} and T_{λ_m} represent temperature of the electric field wave and of the corresponding magnetic field wave with equal energy, then $T_{\lambda_m} = T_{\lambda_e} = 2.8978 \times 10^{-3} \times \frac{1}{\lambda_e}$, when $\lambda_e = \frac{1.129996\times10^{-6}}{\lambda_m}$. More explicitly, $T_{\lambda_m} = 2.564434 \times 10^3 \times \lambda_m$. Thus, the formula for temperature of a magnetic field wave with wavelength λ is $T_\lambda = 2.564434 \times 10^3 \times \lambda$. There are two formulas for temperature of an electromagnetic wave obtained through Wien's displacement law; one for an electric field wave and another one for a magnetic field wave.

3. DISCUSSION ON TWO TYPES OF TEMPERATURE

Two ways to define temperature of electromagnetic waves have been proposed in terms of mean displacement law and in terms of Wien's displacement law. It is difficult to decide about most suitable definition for theoretical development in basic physics. Let us recall that the number $1.063022744 \times 10^{-3}$ m was obtained when the Wien's constant b was divided by the temperature of cosmic microwave background, 2.726 Kelvin. That is, if mode is considered as a representation for Planck's distribution, then every electromagnetic wave for which wavelength is less than $1.063022744 \times 10^{-3}$ m contains only electric field component and every electromagnetic wave for which wavelength is greater than $1.063022744 \times 10^{-3}$ m contains only magnetic field component.

Let us now give importance to mean displacement law and let us divide the number 5.3288843×10^{-3} metre Kelvin by the temperature cosmic microwave background, 2.726 Kelvin, to obtain the number 1.9548365×10^{-3} m. So, if mean is considered as a representation for Planck's distribution, then every electromagnetic wave for which wavelength is less than 1.9548365×10^{-3} m contains only electric field component and every electromagnetic wave for which wavelength is greater than 1.9548365×10^{-3} m contains only magnetic field component. Correctness between these two numbers can be checked only by practical experiments, and conclusion will follow. Correctness between these two numbers will also decide about a good proposal for definition of temperature of electromagnetic waves.

When there is a need to give importance to mean for representation of Planck's distribution, it also changes Planck's constant for magnetic field waves in terms of the relation $\frac{hc}{1.9548365 \times 10^{-3}} = ⓗ \times 1.9548365 \times 10^{-3}$ so that the new Planck's constant for electromagnetic waves is $5.1981367 \times 10^{-20}$. In this case, the formula $\lambda_e = \frac{1.129996 \times 10^{-6}}{\lambda_m}$ given Section 3 becomes $\lambda_e = \frac{3.8213857 \times 10^{-6}}{\lambda_m}$, and hence the formula for temperature of electromagnetic waves $T_{\lambda_m} = 4.7158435 \times 10^3 \times \lambda_m$ becomes $T_{\lambda_m} = 1.39449 \times 10^3 \times \lambda_m$. Thus, the formula for temperature of a magnetic field wave with wavelength λ is $T_\lambda = 1.39449 \times 10^3 \times \lambda$. These changes should be considered while importance to mean for representation of Planck's distribution is given.

4. CONCLUSION

The following two statements are true for two types of temperature of electromagnetic waves. Temperature of an electric field wave increases with decrease in wavelength. Temperature of a magnetic field wave increases with increase in wavelength. The electric field waves of the sun, which are gamma rays, X-rays, visible light rays and non-visible light rays, heat all bodies of our earth, because all bodies have electric fields inside them. Gas movements on the sun may produce magnetic field waves and these waves heat all living beings on the world, because of magnetic fields inside them caused by blood flows. These magnetic field waves may not heat clothes over the bodies of living beings, but they may penetrate clothes and then they may heat inside of bodies.

When there is a magnetic fluctuation over the sun, it is safe for us to be inside a building that may not be penetrated by magnetic field waves of the sun. There is a need to search for a good definition for temperature of electromagnetic waves and this article also proposed two definitions. Let us continue our search. Most interesting finding of this article is that mean distribution law is similar to the Wien's distribution law in forms. One more finding from these forms is that the mean of a Planck's distribution is always greater than the mode of that distribution, when temperature of the black body is fixed.

Authors' contributions

The first author evaluated the two integrals involved in defining the mean displacement law and the second author verified interpretations for concepts related to Planck's distribution.

Informed consent

Not applicable.

Ethical approval

Not applicable.

Conflicts of interests

The authors declare that there are no conflicts of interests.

Funding

The study has not received any external funding.

Data and materials availability

All data associated with this study are present in the paper.

REFERENCES AND NOTES

1. Moorthy CG, Sankar GU. Planck's distribution and definition for temperature of electromagnetic waves. World Sci News 2023; 181:18-31.
2. Moorthy CG. Applications of second order derivative of Planck distribution to cosmic microwave background and melting point. World Sci News 2019; 135:283-88.
3. Planck M. Ueber das gesetz der energieverteilung im normalspektrum. Ann Phys 1901; 309:553–63.