ANALYSIS

55(284), August, 2019

ISSN 2278–5469
EISSN 2278–5450

DISCOVERY

# Comparison between actual and estimated maximum downwind distance using different dispersion parameters

**Khaled SM Essa[1], Aziz N Mina[3], Hany S Hamdy[3], Fawzia Mubarak[2], Ayman A khalifa[3]**

[1]Department of Mathematics and theoretical physics, Nuclear Research Centre, Atomic energy Authority, Cairo, Egypt
[2]Rad. Protection Dept., NRC, AEA, Cairo.
[3]Faculty of science, Beni -Suef University, Egypt

**Article History**
Received: 28 May 2019
Reviewed: 02/June/2019 *to* 30/June/2019
Accepted: 04 July 2019
Prepared: 09 July 2019
Published: August 2019

**Citation**
Khaled SM Essa, Aziz N Mina, Hany S Hamdy, Fawzia Mubarak, Ayman A khalifa. Comparison between actual and estimated maximum downwind distance using different dispersion parameters. *Discovery*, 2019, 55(284), 397-403



## ABSTRACT

In this paper, the maximum downwind distance $x_{max}$ is calculated which depends on the dispersion parameters $\sigma_y$ and $\sigma_z$ by using two methods as follows: the power law function method, and standard method at three effective heights at 5, 100 and 250m respectively. The maximum ground level concentration of air pollution that based on the Gaussian plume model can be calculated by using the values of the estimated and actual $x_{max}$ in the equation of Gaussian air pollution model for different stability classes. The estimated and actual $x_{max}$ are summarized in power law and standard method. Also the percentage of the error between estimated and actual maximum downwind distance are in the range of 0 to 492 and 0.005 to 14.2 respectively. Comparison between the results of estimated and actual $x_{max}$ in case of power law and standard method with the previous work used Brigg's method (Ronbanchob Apiratikul, 2015) are done.

**Keywords**: Gaussian model; maximum downwind distance; maximum concentration; dispersion parameters.

## 1. INTRODUCTION

The concentration of pollutant resulting from a source or group of sources is the main factor of air dispersion modeling under various meteorological conditions. These models are useful for studying the transmission of pollutants into the air (Essa et al., 2015). The concentration of pollutant is a function of number of variables, including the emission rate, the distance of the receptors from the source, and the atmospheric conditions. The atmospheric air quality dispersion model was usually used to estimate the reduction that occurred during the transport of pollutant from an industrial source (wang et al., 2006).

The most common model for studying the air dispersion process was used to improve air quality based on Gaussian plume/ puff formula. The Gaussian model was derived by many authors such as Sutton (1953), Turner (1970) and Smith (1973) was contained many parameters on the basis of empirical correlation$\sigma_y$ and $\sigma_z$ as function of distance were originally developed by pasquill and modified by Gifford. The lateral and vertical dispersion parameter, respectively $\sigma_y$ and $\sigma_z$ represented the turbulent parameterization in this approach. They contained the physical ingredients that described the dispersion process (Abdel Wahab et al., 2013).

In this paper, the maximum concentration at ground level in center-line is predicted with the maximum downwind distance by using the dispersion parameters by two different methods. The Power-law function was the most commonly used method in which plume dispersion coefficients was expressed in terms of downwind distance and atmospheric stability (Sharan et al., 1995). Also the standard method which was based on a single atmospheric stability determined by vertical temperature gradient also the analytical expressions based on (P- G) curves (Essa, et al., 2005). A Comparison between the results of estimated and actual $x_{max}$ in the case of the power law and standard method with the previous work used Brigg's method (Ronbanchob Apiratikul, 2015) were done.

## 2. MATHEMATICAL MODEL

The equation of Gaussian air pollution model can be expressed as equation (1) (IAEA safety guide, 1983):

$$c(x,y,z,H) = \frac{q_p}{2\pi\sigma_y\sigma_z u} e^{-\frac{1}{2}\left[\frac{y}{\sigma_y}\right]^2} \left[e^{-\frac{1}{2}\left[\frac{z-H}{\sigma_z}\right]^2} + e^{-\frac{1}{2}\left[\frac{z+H}{\sigma_z}\right]^2}\right] \qquad (1)$$

where C(x, y, z, H) is the concentration in the downwind distance, crosswind and vertical distance at stack height H, $q_p$ is the emission rate at point source "p",$\sigma_y$ , $\sigma_z$ are crosswind and vertical dispersion parameters respectively, u is the wind speed (m/s).

Equation (1) can be reduced to simple term at the ground level in the plume centerline as follows:

$$c(x,0,0,H) = \frac{q_p}{\pi\sigma_y\sigma_z u}\left[e^{-\frac{1}{2}\left[\frac{H}{\sigma_z}\right]^2}\right] \qquad (2)$$

where the variables $\sigma_y$ , $\sigma_z$ depends on the atmospheric stability classes. The maximum downwind distance and concentration can be derived from the principle of rough estimation by solving equation $\sigma_z = \frac{H}{\sqrt{2}}$ (Kenneth and Cecil, 1972) using estimated and actual methods, so we used two different methods such as power law and standard methods as follows:

1. Power-law functions.

The variables $\sigma_y$ and $\sigma_z$ can be calculated as follows (Essa et al. 2006):

$$\sigma_{y=ax^b} \qquad (4)$$

$$\sigma_{z=cx^d} \qquad (5)$$

where a, b, c, d are constants values depending on the stability classes (Table 1) (Henderson, 1989).

**Table 1** Constants for calculating lateral ( 〖σ〗 _y) and vertical dispersion parameter ( 〖σ〗 _z

| | $\sigma_y\sigma_z$ | | | |
|---|---|---|---|---|
| Stability | a | B | c | D |
| Very unstable | 0.40 | 0.91 | 0.41 | 0.91 |
| Unstable | 0.36 | 0.86 | 0.33 | 0.86 |
| Neutral | 0.32 | 0.78 | 0.22 | 0.78 |
| Stable | 0.31 | 0.71 | 0.06 | 0.71 |

2. Standard method

This method is based on a single atmospheric stability which determined by the vertical temperature gradient, $\frac{\Delta T}{\Delta Z}$ (Green et al., 1980) and the dispersion parameters have the form:

$$\sigma_{y=\frac{rx}{(1+\frac{x}{a})^p}}(5)\sigma_z = \frac{sx}{(1+\frac{x}{a})^q} \qquad (6)$$

where r, s, a, p and q are constants depends on the atmospheric stability (Table 2) (Sharan et al. 1995).

**Table 2** A correspondence between$\frac{\Delta T}{\Delta Z'}$, $\sigma_\theta$ , dispersion parameters and Pasquill stability classes

| Pasquill classes | A | B | C | D | E |
|---|---|---|---|---|---|
| $\frac{\Delta T}{\Delta Z}(\frac{K}{100}\boldsymbol{m}) < -\mathbf{1.9}$ | -1.9to -1.7 | -1.7to -1.5 | -1.5to-0.5 | -0.5to1.5 | |
| $\boldsymbol{\sigma_\theta 25^\circ 20^\circ 15^\circ 10^\circ 5^\circ}$ | | | | | |
| a (km) | 0.927 | 0.370 | 0.283 | 0.707 | 1.07 |
| s (m/km) | 102.0 | 96.2 | 72.2 | 47.5 | 33.5 |
| q | -1.918 | -0.101 | 0.102 | 0.465 | 0.624 |
| r (m/km) | 250 | 202 | 134 | 78.7 | 56.6 |
| p | 0.189 | 0.162 | 0.134 | 0.135 | 0.137 |

Now, one estimates the values of crosswind and vertical standard parameters in power law and standard methods as follows:

**Table 3** power law and standard method for prediction of $\sigma_y$ and $\sigma_z$

| Stability Class | power law $\boldsymbol{\sigma_y}$(m) | $\boldsymbol{\sigma_z}$(m) | standard method $\boldsymbol{\sigma_y(\mathrm{m})\sigma_z(\mathrm{m})}$ |
|---|---|---|---|
| A(very unstable) | $0.4x^{0.91}$ | $0.41x^{0.91}$ | $\frac{0.25x}{1+\frac{0.189x}{0.927}}\frac{0.102x}{1-\frac{1.918x}{0.927}}$ |
| B(moderately unstable) | $0.4x^{0.91}$ | $0.41x^{0.91}$ | $\frac{0.202x}{1+\frac{0.162x}{0.370}}\frac{0.0962x}{1-\frac{0.101x}{0.370}}$ |
| C(slightly unstable) | $0.36x^{0.86}$ | $0.33x^{0.86}$ | $\frac{0.134x}{1+\frac{0.134x}{0.283}}\frac{0.0722x}{1+\frac{0.102x}{0.283}}$ |
| D(neutral) | $0.32x^{0.78}$ | $0.22x^{0.78}$ | $\frac{0.0787x}{1+\frac{0.135x}{0.707}}\frac{0.0475x}{1+\frac{0.465x}{0.707}}$ |
| E(slightly stable) | $0.31x^{0.71}$ | $0.06x^{0.71}$ | $\frac{0.0566x}{1+\frac{0.137x}{1.07}}\frac{0.0335x}{1+\frac{0624x}{1.07}}$ |

## 3. RESULTS AND DISCUSSION

The actual $x_{max}$ of the Gaussian model is determined using programs Mathmatica5 that finds the relation between the maximum ground level concentration and downwind distance through three effective heights at 5, 100 and 250 m were evaluated using $q_p$ and u at 3 g/s and 3 m/s respectively. The percentage of relative error was calculated by

Relative error= $\left|\frac{actual x_{max} - estimated\ x_{max}}{actual\ x_{max}}\right|$

The calculation of $x_{max}$ which depends on the dispersion parameters can be estimated from Table 4

**Table 4** calculation of $x_{max}$

| $x_{max}x_{max}$ Stability class | power law | standard method |
|---|---|---|
| A | $\left(\frac{H}{0.41\sqrt{2}}\right)^{\frac{1}{0.91}}$ | $\frac{H}{0.144+2.07H}$ |
| B | $\left(\frac{H}{0.41\sqrt{2}}\right)^{\frac{1}{0.91}}$ | $\frac{H}{0.136+0.27H}$ |
| C | $\left(\frac{H}{0.33\sqrt{2}}\right)^{\frac{1}{0.86}}$ | $\frac{H}{0.102-0.360H}$ |
| D | $\left(\frac{H}{0.22\sqrt{2}}\right)^{\frac{1}{0.78}}$ | $\frac{H}{0.067-0.657H}$ |
| E | $\left(\frac{H}{0.06\sqrt{2}}\right)^{\frac{1}{0.71}}$ | $\frac{H}{0.047-0.58H}$ |

The prediction of maximum values $x_{max}$ depend on input values which are given in Tables 5 and 6, one concludes that from two Tables, the maximum value of downwind distance depends on stability classes and the effective height of point source, one finds that from the two Tables, there is well agreement between maximum downwind distance in actual and estimated values.

**Table 5** accuracy for the calculation of $x_{max}$ by using power law function

| Stability Class | H=5 Actual $x_{max}$(m) | H=5 Estimated $x_{max}$(m) | H=5 Relative error (%) | H=100 Actual $x_{max}$(m) | H=100 Estimated $x_{max}$(m) | H=100 Relative error (%) | H=250 Actual $x_{max}$(m) | H=250 Estimated $x_{max}$(m) | H=250 Relative error (%) |
|---|---|---|---|---|---|---|---|---|---|
| A | 11 | 11 | 0 | 300 | 287 | 4.33 | 1000 | 786 | 21.4 |
| B | 11 | 11 | 0 | 300 | 287 | 4.33 | 1000 | 786 | 21.4 |
| C | 14 | 16 | 14.3 | 500 | 513 | 2.6 | 2000 | 1489 | 25.55 |
| D | 45 | 35 | 22.22 | 1400 | 1637 | 16.92 | 5000 | 5301 | 6.02 |
| E | 400 | 311 | 22.25 | 15000 | 21174 | 41.16 | 13000 | 76966 | 492 |

**Table 6** accuracy for the calculation of $x_{max}$ by a standard method

| Stability Class | H=5 Actual $x_{max}$ (m) | H=5 Estimated $x_{max}$(m) | H=5 Relative error (%) | H=100 Actual $x_{max}$(m) | H=100 Estimated $x_{max}$(m) | H=100 Relative error (%) | H=250 Actual $x_{max}$(m) | H=250 Estimated $x_{max}$(m) | H=250 Relative error (%) |
|---|---|---|---|---|---|---|---|---|---|
| A | 0.482 | 0.476463 | 1.15 | 0.483 | 0.482756 | 0.05 | 0.483 | 0.482957 | 0.005 |
| B | 3.50 | 3.34225 | 4.5 | 3.64 | 3.65818 | 0.5 | 3.66 | 3.66913 | 0.25 |
| C | -2.90 | -2.94464 | 1.54 | -2.78 | -2.78567 | 0.2 | -2.78 | -2.78093 | 0.03 |
| D | -1.51 | -1.55376 | 2.9 | -1.52 | -1.52362 | 0.24 | -1.52 | -1.52269 | 0.17 |
| E | -1.71 | -1.75254 | 14.2 | -1.71 | -1.725 | 0.87 | -1.71 | -1.7247 | 0.85 |

In power law and standard method, one finds that the percentage of the error between estimated and actual maximum downwind distance in the range of 0 to 492 and 0.005 to 14.2 respectively. This work shows that when we used the method of the power law and standard method are better than Brigg's model method with respect to the previous relative error (Ronbanchob Apiratikul 2015). Then one can deduce that the best values for the estimation $x_{max}$ be observed at the lower $x_{max}$ value.

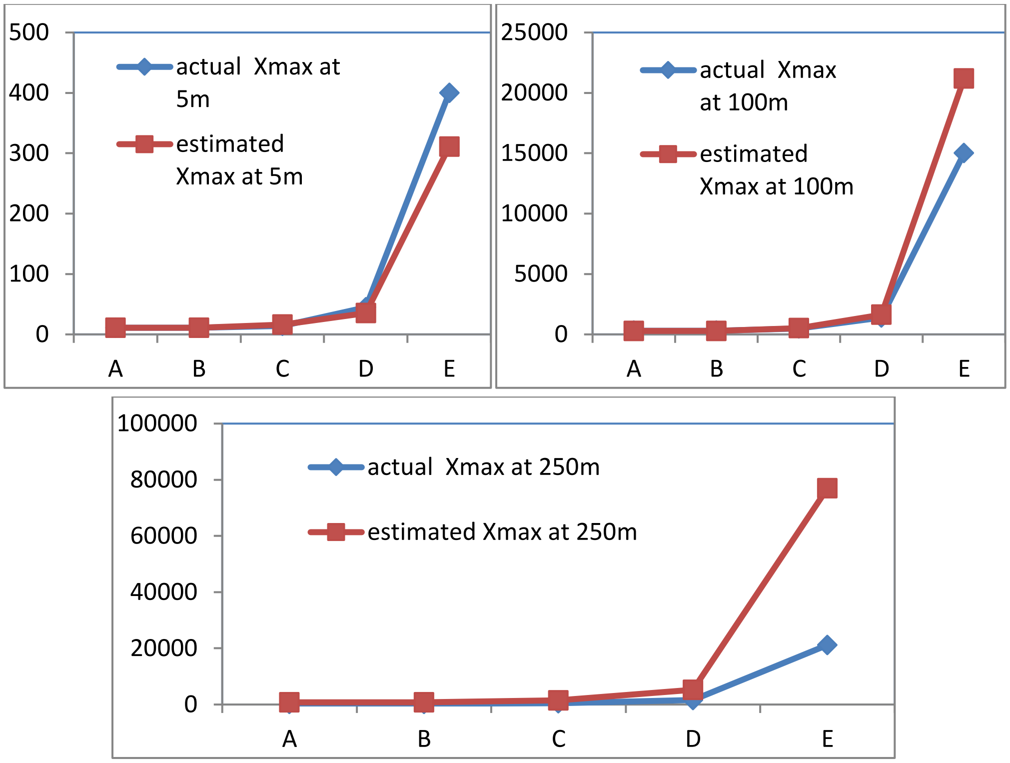


**Figure 1** The variation of the maximum estimated and actual downwind distance (m) using power law

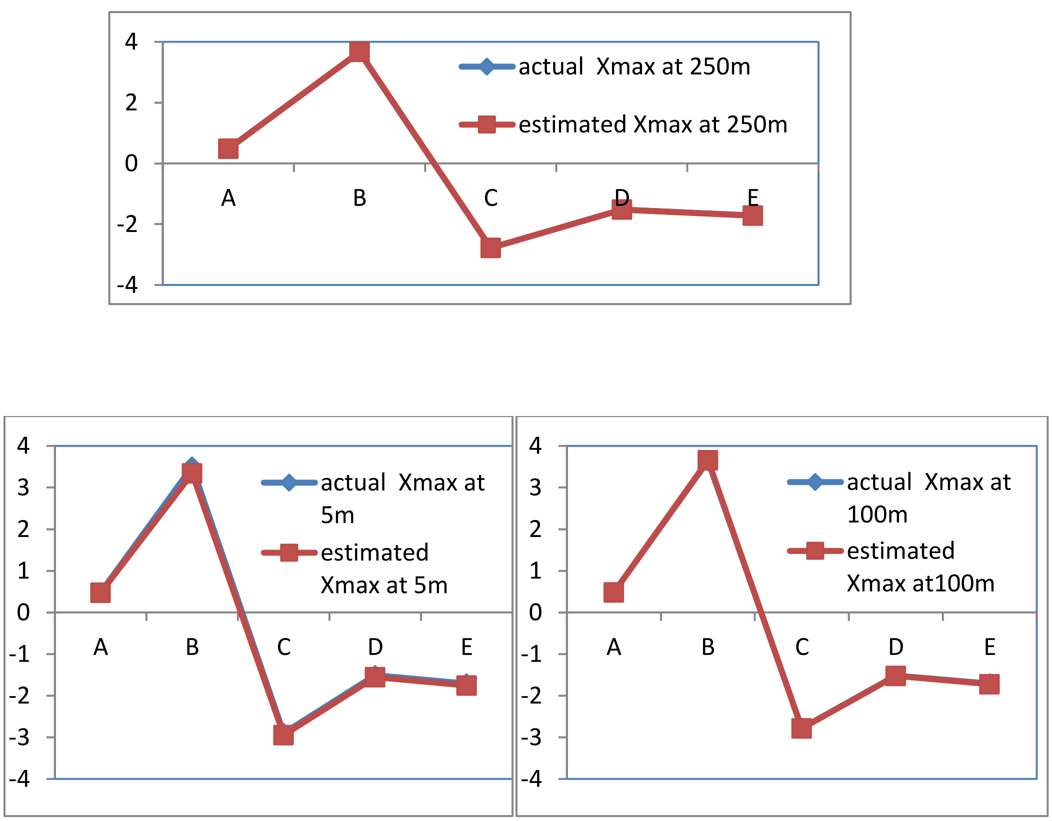


**Figure 2** The variation of the maximum estimated and actual downwind distance (m) using the standard method.

One finds that the two figures show that the variation of the maximum estimated and actual downwind distance in power law and standard methods. From these figures, we conclude that the maximum estimated and actual downwind distances using standard method are better than using power law method but the power law method is a good agreement in all stabilities except stable condition.

**Table 7** shows Accuracy of the calculation of $C_{max}$ by using power law method

| Stability Class | H=5 Actual$c_{max}$1 $0^{-4}(g/m^{-3})$ | H=5 Estimated $C_{max}$ $10^{-4}$ $(g/m^3)$ | H=5 Relative error(%) | H=100 Actual $C_{max}$ $10^{-4}(g/m^3)$ | H=100 Estimated $C_{max}$ $10^{-4}(g/m^3)$ | H=100 Relative error(%) | H=250 Actual $C_{max}$ $10^{-4}(g/m^3)$ | H=250 Estimated $C_{max}$ $10^{-4}(g/m^3)$ | H=250 Relative error(%) |
|---|---|---|---|---|---|---|---|---|---|
| A | 96 | 96 | 0 | 0.24 | 0.24 | 0 | 0.035 | 0.035 | 0 |
| B | 96 | 96 | 0 | 0.24 | 0.24 | 0 | 0.035 | 0.035 | 0 |
| C | 84 | 85.8 | 2.14 | 0.21 | 0.21 | 0 | 0.0307 | 0.03 | 2.28 |
| D | 60.3 | 64.37 | 6.74 | 0.1608 | 0.1609 | 0.06 | 0.0256 | 0.0256 | 0 |
| E | 17 | 18.12 | 6.58 | 0.039 | 0.0453 | 15.38 | $9.21*10^{-7}$ | $9.21*10^{-7}$ | 0 |

**Table 8** shows the accuracy of the calculation of $C_{max}$ standard method

| Stability Class | H=100 Actual $C_{max}10^{-4}(g/m^3)$ | H=5 Estimated $C_{max}10^{-4}(g/m^3)$ | H=5 Relative error(%) | H=100 Actual $C_{max}$ $10^{-4}(g/m^3)$ | H=100 Estimated $C_{max}$ $10^{-4}(g/m^3)$ | H=100 Relative error (%) | H=250 Actual $C_{max}$ $10^{-4}(g/m^3)$ | H=250 Estimated $C_{max}$ $10^{-4}(g/m^3)$ | H=250 Relative error (%) |
|---|---|---|---|---|---|---|---|---|---|
| A | 1545.50 | - 2950 | 290 | 159.253 | 42.783 | 73.13 | 1.56611 | 0.35898 | 77 |
| B | 1212.28 | 1249.9 | 3.1 | 38.9136 | 41.563 | 6.8 | 23.627 | -26.807 | 213 |
| C | 368.313 | 324.175 | 11.98 | 16.3842 | 7.3607 | 55.07 | 1.27759 | 0.49106 | 61.5 |
| D | 1625.53 | -1804.4 | 211 | 68.5025 | -8.1635 | 111.9 | 45.7823 | 0.000718 | 99.9 |
| E | 1204.35 | -1603.9 | 233 | 0.010702 | -0.000968 | 109 | $2.7*10^{-29}$ | -5.25 | $1.94*10^{-29}$ |

Tables 7 and 8 show that the estimated of $c_{max}$ which depend on the maximum downwind distance, effective stack height, and stability classes. One finds that the errors are in the range of 0 to 15.38 and 3.1 to 290 for power law function and standard method respectively.

## 4. CONCLUSION

In this study, the dispersion parameter $\sigma_y$ and$\sigma_z$ can be calculated by using two methods the power law and standard methods at three effective height at 5, 100 and 250m respectively to calculate the maximum downwind distance and ground level concentration for the emission of air pollutants at point source based on Gaussian model. One finds that the percentage of the error between estimated and actual maximum downwind distance in the range of 0 to 492% and 0.005 to 14.2% respectively and the error in the previous work (Ronbanchob Apiratikul 2015) in the range of 0 to 2713.3%. This work shows that when we used the method of power law and standard method are better than Brigg's model method with respect to the presented error. Then one can deduce that the best values for the estimation $x_{max}$ be observed at the lowest $x_{max}$ value.

For the maximum concentration, one finds that the errors are in the range of 0 to 15.38% and 3.1 to 290% in power law and standard methods respectively. Also from the figures, we conclude that the maximum estimated and actual downwind distances using standard method are better than in power law method although the power law method is a good agreement in all stabilities except stable condition.

## REFERENCE


1. Essa K. S. M., Fawzia Mubarak and Elsaid S. E. M (2006)." Effect of the plume rise and wind speed on extreme value of air pollutant concentration", Meteorology and Atoms Phys 93, 247-253.
2. Essa KSM and Elsaid SEM (2015): " Estimation of the Maximum Concentration for non-Gaussian Under Using Different Schemes of Dispersion Parameters for Isotopes", J Civil Environ Eng, 5:4
3. Green AES, Singhal RP, Venkateswar R. 1980. "Analytic extensions of the Gaussian plume model", Journal of the air pollution control association 30(7): 773-776.

4. Henderson B. Sellers (1989)." An Analytical Representation for the pasquill-Gifford-Turner $\sigma_z$ curves for elevated source" Atmosphera, 2, p.p. 111-124.
5. IAEA Safety Guide. (1983). IAEA # 50-5G-S3
6. Khaled S. M. Essa, Fawzia Mubarak and Sanaa Abu Khadra, 2005."Comparison of some sigma schemes for estimation of air pollutant dispersion in moderate and low winds"Atmos. Sci. Let. 6: 90 (2005).
7. Kenneth, W., & Cecil, F.W. (1972). Air Pollution Its Original and Control. Adun-Donnelly Publisher, New York.
8. Mohamed M. Abdel Wahab, Khaled S. M. Essa, Mokhtar Embaby, Sawsan E. M. Elsaid. "Derivation the schemes of lateral and vertical dispersion parameters:application in Gaussian plume model" Open Journal of Air Pollution, 2013, 2, 19-24.
9. Ronbanchob Apiratikul, 2015" Approximation formula for prediction of downwind distance that found the maximum ground level concentration of air pollution based on the Gaussian model", Procedia- Social and Behavioral Sciences 197 (2015) 1257 – 1262.
10. Sharan M, Yadav AK, Singh MP. (Contributing Author). 1995." Comparison of sigma schemes for estimation of air pollutant dispersion in the low wind", Atmospheric Environment 29(16):2051-2059.
11. Smith, M. E. (1973). "Recommended guide for the prediction of the dispersion of airborne effluents", ASME -68-31123.
12. Sutton, O. G. (1953). "Micrometeorology. Quarterly Journal of the Royal Meteorological Society. 79(341), 457.
13. Turner D. B. (1970). "Workbook of atmospheric dispersion", U. S. Environment protection Agency, AP 26.
14. Wang J. S., Cheung C. S. Leung C. W., Hung W. T., "Three-dimensional pollutant concentration dispersion of a vehicular exhaust plume in the real atmosphere" Atmospheric Environment, Vol. 40, 2006, pp.484-497.