ANALYSIS

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Discovery

Analysis of Queuing Model: A Case Study of Madonna Model Secondary School, Owerri

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Queuing theory is the mathematical study of waiting lines [12]. The application of Queuing models was carried out in Madonna Model Secondary School, Owerri, Imo State, Nigeria. Specifically, this study attempts to eliminate waiting time and to formulate a suitable model queuing model as a solution in Dining Hall system. The variables measured include arrival rate (λ) and service rate (μ). They were analyzed for simultaneous efficiency in student satisfaction and cost minimization through the use of a multichannel queuing model, which were compared for a number of queue performances. It was discovered that, using a six-server system with (ρ) is 0.4710 that is 47% of busy server which optimize a balance between waiting time and cost of employing more servers. The study recommended that, the management should maintain a six-server model to increase student's satisfaction.

Keywords: Student Dining hall, Multi-Channel Queuing Model, Student, Queue length, waiting time.

INTRODUCTION

"The Madonna Model Secondary School, Owerri (MMSSO) is a co-educational school situated at layout, Alvan-ikoku college land area with Owerri [4]. The order through which student arrive dinner hall attract queuing problem with the minimal servers. Queuing theory is the mathematical study of waiting lines, or queues [12]. The order through which the student of the MMSSO observe their meal in the dining hall attracts waiting line which is cumbersome time consuming and tedious. Thus, the need to consider the problem of waiting line (queue) in the student dining hall, the average number of students arrival per unit time (λ) or inter arrival between two students (1/λ), the average number of students being served per unit time (μ) or service time between two students (1/μ), service channels, length of queue, queue discipline, maximum number allowed in the system and size of the calling source.

Waiting in line (queue) is certain in a lot of service areas [12]. Queuing theory started with research by Erlang when he created models to describe the Copenhagen telephone exchange. The idea of queuing theory can be traced back to the classical work of Erlang in 1900s, however the work of Kendal in 1951 formed the basis for analytical calculations and the naming convention in queues being used today [6]. For many patients or customers, waiting in lines or queuing is annoying [9].Sharma defined queue as a general phenomenon in everyday life [7]. Queues are formed when customers (human or not) demanding service have to wait because their number exceeds the number of servers available at a given time or the facility doesn't work efficiently or takes more than the time prescribed to service a customer. A queuing system, also known as a processing time, entails the following characteristic: the arrival, the queue discipline, the service mechanism and the cost structure [3]. Oladejo and Aligwo in their work analyzed the existing structure then went ahead to formulate a model which on application with system characteristics was found workable in the hospital queuing system [10]. Soon and Cheng present some basics of queuing theory that instructors may wish to discuss with their pre-university or first-level university students to enable them to partake in the whole modeling process of queues [14]. Tabari et al. in their work concluded that multi-server queuing analysis can be used to estimate the average waiting time, queue lengths, number of servers and service rates [15].

Queuing psychology recognizes that the student's cost of waiting is not just about the time students spend waiting in line, but includes what students think about the waiting. Some customers wait when the total number of customers requiring service exceeds the number of service facilities, some service facilities stand idle when the total number of service facilities

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exceeds the number of customers requiring service [1]. Queuing analysis performed during the initial design of a production facility for electromechanical devices [5].

Queuing theory utilizes mathematical models and performance measures to assess and hopefully improve the flow of customers through a queuing system [11]. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted [12]. [2] Defines queue as simply a waiting line, while [8], put it in similar way as a waiting line by two important elements: the population source of customer from which they can draw and the service system. The population of customer could be finite or infinite [13], many restaurant chains and fast food industry outlets use waiting time standards as an explicitly advertised competitive edge.

Statement of Problem

Based on their scheduled programs, all students are expected to go for their meal at the same time which could result in population upsurge and waiting line in the dining hall. As a consequence of queue length, the total time student spent in queue plus the service time is also affected. In a bid to meet their programs it could or cause problem like balking, reneging, collusion or jockeying.

Direction of current effort

The current effort attempts to minimize the waiting time in dining hall and observe optimal first come first serve order. It then proposes a suitable queue model that minimizes queue length in dining hall system which will improve dining hall service facility.

DATA ANALYSIS & RESULTS

Microsoft-office plus. 2013(excel solver) was used for the computation, Analysis and summary of results of the data are presented and discussed

The queuing model used in the analysis is *M/M/s* which involves a single-line with multiple servers in the system.

The following assumptions are made:

1. The students face balking, reneging, or jockeying and come from a population that can be considered as infinite.
2. Student arrivals are described by a Poisson distribution with a mean arrival rate of λ (lambda). This means that the time between successive student arrivals follows an exponential distribution with an average of 1/ λ.
3. The student service rate is described by a Poisson distribution with a mean service rate of μ (mμ). This means that the service time for one student follows an exponential distribution with an average of 1/μ.
4. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system.

Source of data

The data was collected from Madonna model secondary school, Owerri at different days and times during breakfast, lunch and dinner which involves arrival and service time of students. The data was collected within some randomly selected meals, so as to check whether students face the same situation at any time they enter the dining hall for their meal. The collection was based on the number of student's arrival time and service time. The data was collected with an average of one hour during the days of 14th, 16th, 25th and 29th all in the month of October, 17th, 21st, 28th were also for the month of November and 2nd December all in year 2017 (Table 1). Results for sample computation are shown below for the data of 14th October, The results for other dates follow similar calculations using excel solver (Table 2).

1. Utilization factor for 14th October is given by: $\rho = p = \frac{\lambda}{s\mu} = \frac{520}{3\times187} = 0.9269$

2. The probability that at any given time the system will be idle (there are no students waiting).

$$P_0 = \left(\sum_{j=0}^{s-1} \frac{(sp)^j}{j!} + \frac{(sp)^s}{s!\,(1-p)}\right)^{-1}$$

$$= \left(\sum_{j=0}^{3-1} \frac{(3 \times 0.9269)^j}{j!} + \frac{(3 \times 0.9269)^6}{3!\,(1- 0.9269)}\right)^{-1}$$

$$= \left(\frac{(3\times 0.9269)^0}{0!} + \frac{(3\times 0.9269)^1}{1!} + \frac{(3\times 0.9269)^2}{2!} + \frac{(3\times 0.9269)^3}{3!(1-0.9269)}\right)^{-1}$$

$$P_0 = (1 + 2.7807 + 3.8661 + 49.0223)^{-1}$$

$$P_0 = 0.017642$$

3. The average number of students waiting in queue to be served L_q

$$L_q = P_0 \frac{S^s p^{s+1}}{S!\,(1-p)^2}$$

$$= 0.017642 \times \frac{3^3(0.9269)^{3+1}}{3!\,(1-0.9269)^2}$$

$$L_q = 10.9719$$

4. The average number of students in the servers Ls

$$Ls = \frac{\lambda}{\mu} = \frac{520}{187} = 2.7807$$

5. The average number of students in the system L

$$L = L_q + Ls$$

$$= 10.9719 + 2.7807$$

$$L = 13.7526$$

6. The average time a student spend in waiting in queue before service starts Wq is

$$W_q = \frac{L_q}{\lambda}$$

$$= \frac{2.921348}{520} = 0.0211$$

7. The average time students are served Ws is

$$Ws = \frac{1}{\mu}$$

$$= \frac{1}{187} = 0.005348$$

8. The average time a student spends in the system, waiting plus served

$$W = W_q + \frac{1}{\mu} \quad but\ W_q = \frac{L_q}{\lambda}$$

$$W = 0.0211 + 0.005348 = 0.02644$$

Table 1 Shows Primary Data Summary for the Randomly Selected Hours and Days

Date	Time range	Arrival Rate	No. of Servers	Service Rate
14th Oct	6:45am – 7:45am	520	3	187
16th Oct	3:00pm – 4:00pm	980	3	339
25th Oct	06:45am-07:45am	650	3	230
29th Oct	3:00pm – 4:00pm	720	3	253
17th Nov	3:00pm – 4:00pm	752	3	265
21st Nov	3:00pm – 4:00pm	780	3	273
28th Nov	6:45am – 7:45am	680	3	244
2nd Dec	7:00pm – 8:00pm	580	3	206

Table 2

Date	Oct_14	Oct _16	Oct _25	Oct _29	Nov_17	Nov_21	Nov_28	Dec_2
Arrival Rate	520	980	650	720	752	780	680	580
Service Rate/Channel	187	339	230	253	265	273	244	206
Number of Servers	3	3	3	3	3	3	3	3
Type	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3
Mean Number at Station (L)	13.753	27.577	17.329	19.544	18.569	21.085	14.148	16.340
Mean Time at Station (W)	0.0264	0.0281	0.0267	0.0271	0.0247	0.0270	0.0208	0.0282
Mean Number in Queue (Lq)	10.972	24.687	14.502	16.698	15.731	18.227	11.361	13.524
Mean Time in Queue (Wq)	0.0211	0.0252	0.0223	0.0232	0.0209	0.0234	0.0167	0.0233
Mean Number in Service (Ls)	2.7807	2.8908	2.8261	2.8459	2.8377	2.8571	2.7869	2.8155
Mean Time in Service (Ws)	0.0054	0.0030	0.0045	0.0040	0.0038	0.0037	0.0041	0.0049
Efficiency (ρ)	0.9269	0.9636	0.9420	0.9486	0.9459	0.9524	0.9290	0.9385
Probability All Servers Idle (ρ_o)	0.0176	0.0084	0.0138	0.0121	0.0128	0.0112	0.0171	0.0146
Prob. All Servers Busy	0.8651	0.9320	0.8925	0.9045	0.8995	0.9114	0.8688	0.8861

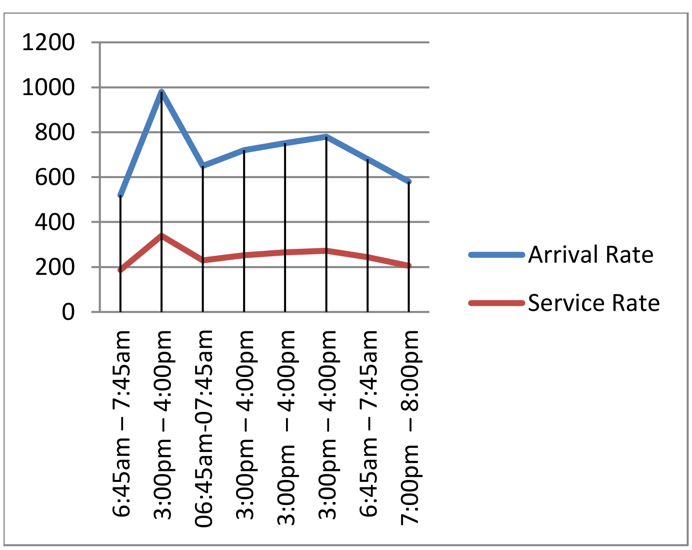

Figure 1 Population upsurge

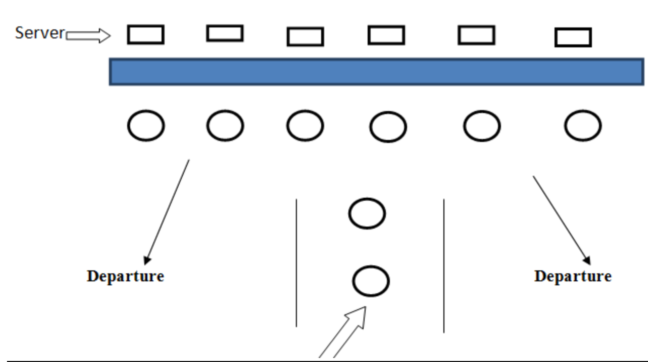

Figure 2 Multi-channel queuing model with 6 channels

Table 3 Shows Results of Types Models from One to Eight Servers at a Given Point

Arrival Rate	650	650	650	650	650	650	650	650
Service Rate/Channel	230	230	230	230	230	230	230	230
Number of Servers	1	2	3	4	5	6	7	8
Type	M/M/1	M/M2	M/M/3	M/M/4	M/M/5	M/M/6	M/M/7	M/M/8
Mean No. at Station(*L*)	-1.5476	2.1627	17.3286	3.8825	3.0799	2.8958	2.8453	2.8311
Mean Time at Station(W)	-0.0024	0.0033	0.0267	0.0060	0.0047	0.0045	0.0044	0.0044
Mean No. in Queue(Lq)	-4.3737	-0.6632	14.5025	1.0564	0.2539	0.0697	0.0192	0.0050
Mean Time in Queue(*Wq*)	-0.0067	-0.0010	0.0223	0.0016	0.0004	0.0001	2.95E-05	7.76E-06
Mean No. in Service(Ls)	2.8261	2.8261	2.8261	2.8261	2.8261	2.8261	2.8261	2.8261
Mean Time in Service(*Ws*)	0.0043	0.0043	0.0043	0.0044	0.0043	0.0043	0.0043	0.0043
Efficiency(ρ)	2.8261	1.4130	0.9420	0.7065	0.5652	0.4710	0.4037	0.3533
Prob. All Servers Idle(ρ_o)	-1.8261	-0.1712	0.0138	0.0485	0.0565	0.0586	0.0591	0.0592

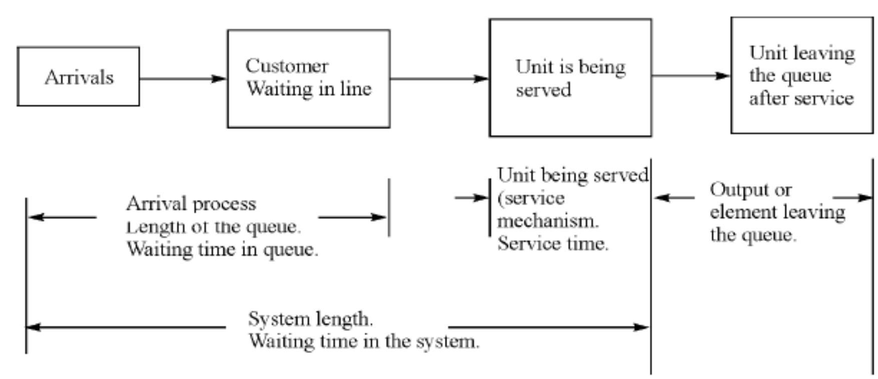

Figure 3 The queuing system components

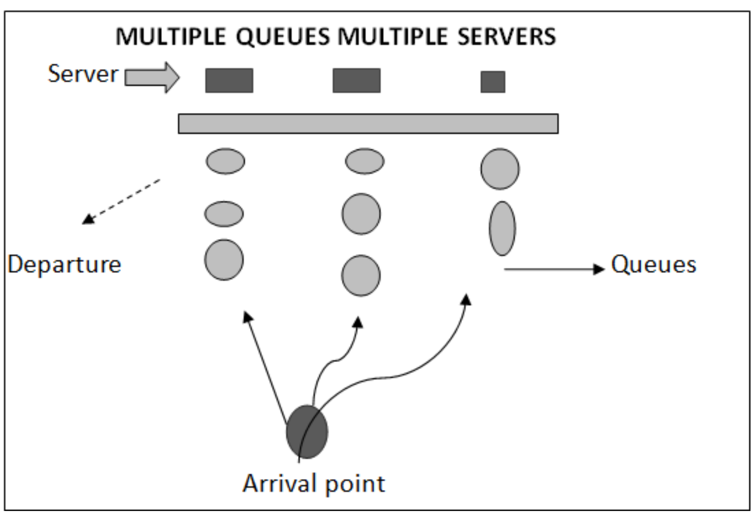

Figure 4 Extant Queuing Structure of MMSSO Student Dining hall

Projections using 25th October 2017

Let us consider one of the days in which student recorded capacity utilization closer to the total average service utilization. Use an arrival rate, (λ) and service rate, (μ) for the analysis (Table 3). The graph shows that they have population upsurge while dinner is the lowest during lunch, as well as service rate (Figure 1). The average number of students waiting in the system and time they are served remain constant from one-server to eight-servers. The inappropriateness of a single server model for solving students – waiting time problems become apparent as it shows negative figures for all performance criteria accept (ρ), (Ls) and (Ws). However, multi – server models were compared and it is seen that;

- Using a six – server system with (ρ) is 0.4710 that is 47% of busy server which optimize both the waiting time and cost, that is, to strike a balance between waiting time and cost of employing more servers. Using a six – server system is better than a three – server system in all complicating result. For instance, assuming during that morning, there were three servers serving the students, there would have been 17.3286 students in queue system instead of 2.8958 students and the time spend in system is 0.0267 instead of 0.0045 hours respectively.
- A six – server system has a high probability of being idle 0.0586 than five – server, four – server and three server system.

The proposed model

On analysis, this model proved workable as it produced the desired result of reducing queuing time. It is therefore presented here as the proposed model. ***M/M/6/FCFS/∞/∞*** **.**This is a multi-channel queuing model with 6 channels, arrival and service times are both Poisson. The queue discipline is first come first serve. It has one queue from which student are allocated to the channels (Figure 2).

DISCUSSION

The results show that the server would be busy 94.34% of the time and idle 1.35% of the time. Also, the average number of students in the queue is 16 and the average number of students in the system is 19. More so, the average time a student spends in the queue is 0.0220 hours and average a student spends in the system is 0.0261 hours. It is determined using six-servers that students spent little time at the student dining hall system of MMSSO. From the result obtained, a student spent an average of 0.0261 hours that is 1.6 minute in the system. During their meal, they spend an average of 0.0246, 0.0267 and 0.0282hours for breakfast, lunch and dinner respectively. 2nd December recorded the highest waiting time spent in dining hall system with 0.0282hour and it was followed by 16th October with 0.0281 students as compared to 17th November and 28th November had the least waiting time in the system with 0.0270 and 0.0208 hours respectively as shown in table 2. It is also observed that students waiting line (queuing length) is much at student dining hall of MMSSO if still using three-server system. The average number of students in the system from Table3 is 19 students will be in the system. For 16th October recorded the highest number of students the dining hall which is 28 students and it was followed by 21st of November that is 21 students.

CONCLUSION

This study minimizes the amount of waiting time a student is likely to experience and thus reduce population upsurge in dining hall system. It was determined using six-servers from Table 3; a student will spend 0.0045hour (16 sec.) waiting time in dining hall system, whereas using three-servers system will be 0.0267 (1min, 60sec). It was determined using six-servers from Table 3; they will be 3 students in queue system, whereas using three-servers will upsurge to 19 students on the average.

MATERIAL AND METHODS

The queuing system consists essentially of three major components (Figure 3):

(a) The source population and the way students arrive at the system,

(b) The servicing system, and

(c) The condition of the students exiting the system.

The system consist of more servers, an arrival pattern of student, service pattern, queue discipline, the order in which services are provided and student behavior.

Multiple Queue and Multiple Servers

This can also be called Single Stag Queue in parallel as described in Figure 4. It is similar to that of Single Queue – Server Queue, only that there are many servers performing the same task with each having a queue to be served. This type of queue is practiced in Madonna Model Secondary School, Dining hall, Owerri.

Method

a. Little's law

According to Little (1961), The long-term average number of customers in a stable system L, is equal to the long-term average arrival rate, λ, multiplied by the long-term average time a customer spends in the system, W; i.e L = λw

b. Notation for queues

Since all queues are characterized by arrival, service and queue and its discipline, the queue system is usually described in shorten form by using the D. G kendall notation [16]:

{A/B/S}:{d/e/f}

Where,

A= probability distribution of the arrivals

B= probability distribution of the departures

S = number of servers (channels)

d = the capacity of the queue(s)

e = the size of the calling population

f = queue ranking rule (ordering of the queue).

c. M/M/s model

The description of a M/M/s queue is similar to that of the classic M/M/1 queue with the exception that there are s servers. When s=1, all the result for the M/M/1 queue can be obtained. The number of students in the system at time t, x(t), in the M/M/s queue can be modeled as a continuous times Markov chain.

The condition for stability is $p = \frac{\lambda}{s\mu} < 1$ where λ is mean arrival rate, μ is mean service rate, s is number of servers and p is called the service utilization factor or the proportion of time on average that each server is busy. The total service rate must be greater than the arrival rate, that is $s\mu > \lambda$, and if $s\mu \leq \lambda$ the queue would eventually grow infinitely large.

(1) The probability that at any given time there are no students waiting or being served at steady state

$$P_0 = \left(\sum_{j=0}^{s-1} \frac{(sp)^j}{j!} + \frac{(sp)^s}{s!\,(1-p)} \right)^{-1}$$

Where:
S = number of servers
p = service utilization factor
j = range of server (s) for j = 0, 1, 2, ... , s-1

(2) The average number of students waiting in queue to be served L_q.

$$L_q = P_0 \frac{s^s p^{s+1}}{S!\,(1-p)^2}$$

(3) The average number of students in service L_s,

$$L_s = \sum_{j-1}^{s-1} jP_j + \sum_{j=s}^{\infty} sP_j = sp$$

(4) The average number of students in the system becomes

$$L = L_q + L_s = L_q + sp = L_q + \frac{\lambda}{\mu}$$

(5) The average time student spend in waiting in queue before service starts W_q is

$$W_q = \frac{L_a}{\lambda}$$

(6) The average time student spend in the system, waiting plus being served W is

$$W = \frac{L}{\lambda} \frac{L_a + \frac{\lambda}{\mu}}{\lambda} = \frac{L_a}{\lambda} + \frac{1}{\lambda} = W_q + \frac{1}{\mu}$$

(7) The average time students are served $W_s = W - W_q = \left(W_q + \frac{1}{\mu}\right) - W_q = \frac{1}{\mu}$

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