Discovery

The golden ratio breaks

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ABSTRACT

Based on El Naschie's relation ($\alpha^{-1} = 20/\phi^4$), it was found that the golden ratio is slightly broken at subatomic scale. That takes significant physical meanings. However it is nothing if its energy was as high as about the magnitude of MeV; there are almost no consequence are caused, otherwise as energy down lowed at or below KeV, the symmetry would break at all. Also because of the broken, that the multidimensional real physical space above four dimensions was proved exist in subatomic scale.

1. INTRODUCTION

After the tremendous success of special relativity, general relativity, and quantum mechanics established in early twenty century, at micro level the focus of physics has removed from atom scale onto the followed subshell research—field of elementary particle physics. The harvests of standard model and afterward superstring theory become the landmarks in the field since 1964 (by Murray Gell-Mann, Peter M. Higgs et al.) and 1969 (by Yoichiro Nambu, Holger Bech Nielsen, Leonard Susskind, and Edward Witten in 1990).

It must be motioned that in the period, a remarkable work has been contributed by M.S. El Naschie since 1992, and following Wolfgang Seelig in 1997. In attempt to calculate the mass spectrum of elementary particles, they all gave out a good result by a simplest formula. The key point is that, their resultant formulas are concerned with the inverse fine structure constant. More amazingly, giant El Naschie related mass spectrum with mathematically the golden mean (El Naschie, 2002a, b; Marek-Crnjac, 2003). It takes great meanings. Because the golden ratio is a special number, it represents perfect symmetries in constituting structures in a way. However, by carefully examining we find that the golden ratio breaks in field of elementary particle physics. The situations of the breaking are described in this paper.

2. A HISTORY REVIEW ABOUT THE GOLDEN RATIO

a)   The Originsince Definition

Golden ratio—a special and charming mathematical number, was found by ancient Greek mathematicians Fibonacci and Euclid 2400 years ago. Its origin was as follows:

As to none zero number a and b, if meet

$$\frac{a}{a+b} = \frac{b}{a} \equiv \phi \text{, or } \frac{a+b}{a} = \frac{a}{b} \equiv \varphi,$$

Then we call the number ϕ or φ to be the golden ratio or golden mean. ϕ and φ are the infinite non-repeating decimal, its number can simplest expressed as

$$\varphi = 1.6180339887..., \text{ or}$$

$$\phi = 0.6180339887...$$

It can derived as

$$\frac{a+b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\varphi} = \varphi, \quad \varphi^2 = \varphi + 1, \quad \varphi^2 - \varphi - 1 = 0, \text{ solve the equation we have the result}$$

$$\varphi = \frac{\sqrt{5}+1}{2} = 1.6180339887 \text{ ---, as } \phi = \varphi - 1, \text{ we can also have}$$

$$\phi = \frac{\sqrt{5}-1}{2} = 0.6180339887 \text{ ---}$$

b)   The Symmetry Characters

As the golden ratio to be the central point of this paper, I'd like to summarize and represent its properties here in order to feel its beauty, emphasize its importance, and make things to be more clearly. Their magical mathematical relations are list as follows taking the ϕ as the example.

First:

$$\phi = \frac{\sqrt{5}-1}{2} = \frac{a_1\sqrt{5} + (-1)b_1}{2},$$

$$\phi^2 = \frac{-\sqrt{5}+3}{2} = \frac{-a_2\sqrt{5}+b_2}{2},$$

$$\phi^3 = \frac{2\sqrt{5}-4}{2} = \frac{a_3\sqrt{5}-b_3}{2},$$

$$\phi^4 = \frac{-3\sqrt{5}+7}{2} = \frac{-a_4\sqrt{5}+b_4}{2},$$

$$\phi^n = \frac{(-1)^{n-1}a_n\sqrt{5}+(-1)^n b_n}{2}, \quad n \in N, n \geq 3 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ (1)$$

Second:

$$\frac{1}{\phi} = 1+\phi, \quad \frac{1}{\phi^2} = 3-\phi^2, \quad \frac{1}{\phi^3} = 4+\phi^3, \quad \frac{1}{\phi^4} = 7-\phi^4, \quad \frac{1}{\phi^5} = 11+\phi^5, \quad ---,$$

$$\frac{1}{\phi^n} = (b_{n-2}+b_{n-1})+(-1)^{b-1}\phi^n, \quad n \in N, n \geq 3 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ (2)$$

Equation (1) and (2) showed us that the golden ratio has an invariable form in its exponentiation including inverse form.

Third:

$$\phi^{n+1} = \phi^n + \phi^{n-1} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ (3)$$

So, we have

$$\phi^1 = \phi, \quad \phi^2 = 1+\phi, \quad \phi^3 = 1+2\phi, \quad \phi^4 = 2+3\phi, \quad \phi^5 = 3+5\phi, \quad \phi^6 = 5+8\phi, \quad \phi^7 = 8+13\phi, ---,$$

$$\phi^n = a_{n-1}+a_n\phi$$

It shows that any power of ϕ can express linearly by ϕ, and their coefficients are the regularly integers. Where $a_1 = 1$, $a_2 = 1$, $a_3 = 2$, $a_4 = 3$, $a_5 = 5$, $a_6 = 8$,---, $a_n = a_{n-2} + a_{n-1}$; and $b_1 = 1$, $b_2 = 3$, $b_3 = 4$, $b_4 = 7$, $b_5 = 11$, ---, $b_n = b_{n-2} + b_{n-1}$.

Fourth:

$$\frac{1}{\phi} = 2\cos\frac{\pi}{5} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ (4)$$

Equation (4) reveals the golden ratio the geometric means.

All above characters show us the golden ratio's perfect symmetry properties.

c)   The Origin from Series

In thirteen century, Italian mathematician Filius Bonacci point out an infinite series from 1 and 2, and the next number is formed by plus two formal ones. So obtained the series as

$$\begin{cases} 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, \\ 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, \ \ldots \end{cases}$$

From the series, by dividing former two numbers to create the next number, a new series is formed as,

$$\begin{cases} 0.5, 0.66666, 0.6, 0.625, 0.615, 0.619, 0.617, 0.618, 0.617, 0.61805, 0.61802, 0.618037, 0.618032, \\ 0.618034, 0.618033, 0.618034, 0.61803396, 0.61803399, 0.61803398, 0.61803399, 0.618033988, \\ 0.618033989, 0.6180339887, 0.6180339888, 0.6180339887, 0.6180339888, 0.6180339887, \\ 0.6180339888, 0.6180339887, 0.6180339887, 0.6180339887, \ \ldots \end{cases}$$

$$. \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (5)$$

It is clear that the latest number in series (5) is pressed on towards the golden ratio gradually as exact as demand.

3. THE GOLDEN RATIO BREAKS

As it is well known, that golden ratio is operated more as a universal law. It presents in art, music, plants, animals, chemical crystals, and the human body. Even it occurred at the atomic scale, the human genome DNA is connected with it. Moreover, as El Naschie's contribution, we know that may the golden ratio governed the rules at subatomic scale.

El Naschie published a dozens more papers aimed to calculating the mass spectrum of elementary particles. By the use of his Cantorian E-infinity space theory, he successfully predicted the accurate values of mass spectrum of elementary particles. The golden ratio emerges naturally in his theory, and turns out to be the central piece that connects the fractal dimension of quantum space-time with the mass-energy of every fundamental particle, and specially with fundamental physical quantity such as inverse fine structure constant (El Naschie, 2002).

The constant has determined many times by several experimental methods in order to acquire a more accurate value. The recent accurate value suggested in 2010 by CODATA is (刘荣智 et al., 2015).

$$\alpha^{-1}(\text{experimental}) = 137.035999074(44)$$

The inverse fine structure constant α^{-1} was considered to have several physical meaning. In 1916, A. Sommerfeld gave out a theoretical explanation of α^{-1}, such as

$$\alpha^{-1}(\text{theoretical}) = \frac{c}{v_e} = \frac{2\varepsilon_0 hc}{e^2}$$

$$= \frac{2 \times 8.854187817 \times 10^{-12} \times 6.6260755 \times 10^{-34} \times 2.99792458 \times 10^8}{(1.60217733 \times 10^{-19})^2}$$

$$= 137.035990735$$

Where, v_e is the velocity of hydrogen atom electron at ground state. Other variables are held their normal meaning.

Both the experimental and theoretical ones are accurate enough and almost to be equal. Chosen the coincident parts, we can simply express the real inverse fine structure constant in short as

$$\alpha^{-1} = 137.03599 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (6)$$

The important things is that El Naschie found that

$$\frac{20}{\phi^4} = 137.08203 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (7)$$

It takes as similar as the same value with 137.03599. From this enlightenment, El Naschie gave out the inverse fine structure constant's another explanation, that

$$\alpha^{-1} = \frac{20}{\phi^4} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (8)$$

However, they (equation (6) and (7)) are not exactly equal. There are obviously slight deviations from the second decimal place. Why such like this?

I agree with El Naschie's guess or explanation, i.e., the relation of equation (8) ought to be exact correct. Let's rewrite it in a strict form as

$$\alpha^{-1} = \frac{20}{\phi_\square^{4}} \quad \text{.........................}\quad (9)$$

So, we can obtain the value of the new inductive parameter, $\phi_\square$, as

$$\phi_\square = (\frac{20}{\alpha^{-1}})^{\frac{1}{4}} = (\frac{20}{137.03599})^{\frac{1}{4}}$$

$$\phi_\square = 0.6180859030 \quad \text{.........................}\quad (10)$$

For convenient comparison, let's recall the expression of ϕ as

$$\phi = 0.6180339887... \quad \text{.........................}\quad (11)$$

The present difference between ϕ and $\phi_\square$ must mean something. If we insist on the relations in equation (8), it showed us that the golden ratio ϕ must break down to $\phi_\square$. The breaking point is at the fifth decimal place after the number "0.6180".

4. DISCUSSION

d)   Quantum Character

The number "0.6180" correspond to the eleventh number "0.61802" (the first stable number) in series (5). It can be defined a name of broken golden ratio as convenient,

$$\phi_\Omega = 0.61802$$

It means that, in the process of structure forming mechanism in subatomic scale, there is no infinite accuracy for demanding a perfect golden cut; the unit constructing "materials" are quantized. It also showed that the finite accuracy is naturally stopped at eleventh iteration. But, what does the eleven mean?

e)   Symmetry Breaking and Extra-Dimension

As summarized in paragraph 2.2, the golden ratio corresponds to the perfect infinite symmetries. But now the golden ratio breaks, it causes the perfect symmetries breaking accordingly. Though the exact golden ratio is not exist in nature in subatomic scale, the broken golden ratio (ϕ_Ω) has remain some symmetries to some extent. The fantastic extents are right concealed in ϕ_Ω.

The broken golden ratio is resulted after 11-times iteration. According to El Naschie's result (El Naschie, 2002), I guess it corresponds to 11 kinds of symmetry; in terms of $\phi_\Omega^{\,n}$, it can express in row matrix as

$$\left(\phi_\Omega^{\,1}, \phi_\Omega^{\,2}, \phi_\Omega^{\,3}, \phi_\Omega^{\,4}, \phi_\Omega^{\,5}, \phi_\Omega^{\,6}, \phi_\Omega^{\,7}, \phi_\Omega^{\,8}, \phi_\Omega^{\,9}, \phi_\Omega^{\,10}, \phi_\Omega^{\,11} \right) \quad \text{.....................}\quad (12)$$

If you would like to describe the nature in subatomic scale by using the 11 elements in matrix (12), than it means that you are faced and referred to 11-demensional vector space.

11 elements of matrix correspond to 11 kinds of symmetry group, and 11 kinds of symmetry group would in law correspond to 11 regular polytopes in the geometry. As 3-D space has as more as 5 regular polytopes, and 4-D space has as more as 6 regular polytopes, the 11 regular polytopes means that there exists the multidimensional real physical space above four dimensions in subatomic scale.

f) Comparison of ϕ and ϕ_Ω in Mass Spectrum and KeVRange

Although ϕ_Ω has slight deviation compared with ϕ numerically, it express the notable quite different physical meanings. We cansay that the ϕ_Ω stands for the actual golden ratio in subatomic scale, and ϕ is the ideal golden ratio absolutely. In the strict sense, one may substitute ϕ_Ω for ϕ in subatomic scale. Take the mass spectrum calculation as an example; some of their results are list in Table 1.

Table 1 Mass calculation for some elementary particles in terms of ϕ and ϕ_Ω

Particle	Theoretical mass in terms of ϕ [2–4]	Theoretical mass in terms of ϕ_Ω	Experimental value
e (electron)	$\dfrac{1}{\sqrt{10}}(1+\phi) = 0.51167$	$\dfrac{1}{\sqrt{10}}(1+\phi_\Omega) = 0.51167$	0.511 MeV
p (proton)	$\dfrac{(20(7-\phi^4)-\phi^5(1-\phi^5))^2}{20} = 938.45$	$\dfrac{(20(7-\phi_\Omega^4)-\phi_\Omega^5(1-\phi_\Omega^5))^2}{20} = 938.45$	938.27231 MeV
n (neutron)	$20\dfrac{1}{\phi^8} = 939.574$	$20\dfrac{1}{\phi_\Omega^8} = 939.744$	939.563 MeV
ν_e (e neutrino)	$\phi^3(1-\phi^3) = 0.1803399$	$\phi_\Omega^3(1-\phi_\Omega^3) = 0.1803314$	MeV
μ (μ meson)	$10(3-\phi^2)\sqrt{10(1+\phi)} = 105.3099$	$10(3-\phi_\Omega^2)\sqrt{10(1+\phi_\Omega)} = 105.3101$	105.65839 MeV
π (π meson)	$\dfrac{5}{2}(7-\phi^4)(8+\phi^4) = 139.58204$	$\dfrac{5}{2}(7-\phi_\Omega^4)(8+\phi_\Omega^4) = 139.58208$	134.98 MeV
κ (κ meson)	$\dfrac{20}{\phi^4}\left(3+\phi-\dfrac{\phi^4}{10}\right) = 493.97$	$\dfrac{20}{\phi_\Omega^4}\left(3+\phi-\dfrac{\phi_\Omega^4}{10}\right) = 494.01$	493.646 MeV
τ (τ meson)	$99(18-\phi^6) = 1776.4829$	$99(18-\phi_\Omega^6) = 1776.4837$	1777 MeV
J/ψ (meson)	$5\dfrac{1}{\phi^9}(8+\phi^4) = 3095.98$	$5\dfrac{1}{\phi_\Omega^9}(8+\phi_\Omega^4) = 3096.60$	3096.9 MeV
Σ (meson)	$6\dfrac{1}{\phi^{11}} = 1194.030151$	$6\dfrac{1}{\phi_\Omega^{11}} = 1194.3274$	1193.28 MeV
u (up quark)	$5+\phi^3 = 5.236068$	$5+\phi_\Omega^3 = 5.236052$	MeV
d (down quark)	$\dfrac{2}{\phi^3} = 8.472136$	$\dfrac{2}{\phi_\Omega^3} = 8.472711$	MeV
u /d (ratio)	$5.236068/8.472136 = 0.618033988$ $= \phi$	$5.236052/8.472711 = 0.617990157$ $\neq \phi_\Omega$	

As we can see in Table 1 that, because ϕ_Ω and ϕ are very close numerically, the consequence of the substitution is as similar as the same. However, if the energy level was at or below the magnitude of KeV, one can predict the appearance of obvious broken golden ratio; that means the symmetry broke thoroughly, or to say that there are no symmetries at all.

5. CONCLUSION

The golden ratio is broken at subatomic scale, and the symmetries break accordingly. Nevertheless the breakings are very weak; it plays almost no influence to properties when the energy level is about or above MeV. Otherwise, as energy down lowed at or below KeV magnitude, the symmetry would break thoroughly. Also, the broken golden ratio has proved the existence of the multidimensional real physical space above four dimensions in subatomic scale.

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