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Reconstruction of Fundamental Quantum Field Theory for Describing Scalar Field Dynamics in the Early Universe

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ABSTRACT

It is demonstrated that there is some illusive problem in unfolding quantum field theory relevant to the early universe. That is an inadequate formulation of quantum field momentum in previous works, which is the conjugate variable of the scalar field. Most researchers in this field define quantum field momentum as $\hat{\pi}_k = -i\partial/\partial\phi_k$ under the choice $\hbar = c = 1$, where ϕ_k is the scalar field. However, it is proved that this definition in quantum domain is inadequate because its dimension does not match with that of the counterpart classical representation $\pi_k = \partial\mathcal{L}_k/\partial\dot{\phi}_k$ where $\mathcal{L}_k$ is the generalized Langrangian density. We suggest some recipes for fixing this problem on the basis of reconstruction of the quantum field momentum. Quantum field theory with our improved definition of quantum field momentum does not raise dimensional problem and agrees with our previous researches.

Keywords: Quantum Cosmology, Scalar Field, Field Momentum, Inflation Model of the Universe, Hamiltonian Dynamics

Abbreviations: FRW spacetime - Friedmann-Robertson-Walker spacetime

1. INTRODUCTION

The evolution of the early universe, that is usually studied from the Einstein's field equations, is a critical issue in modern cosmology. From numerous reports published in this field, we can see that inflation universe model (Guth, 1981; Guth et al. 1985; Suresh, 2004; Kim et al. 2004; Choi, 2013; Kim, 2007; Bak et al. 1998; Kolb et al. 1990) based on scalar field dynamics is a main stream of researches in cosmology. Inflation model supposes an extremely rapid large scale inflation of the universe at the very early stage of cosmological time, dictated by enormous vacuum energy characterized by the scalar field (Kolb et al. 1990). Recent observations (Blake et al. 2011a; Blake et al. 2011b; Sullivan et al. 2011) of the universe reveal that vacuum energy or cosmological constant is an authentic candidate of dark energy which has been usually regarded as a mysterious ingredient of the universe. As is well known, the dark energy problem (or cosmological constant problem) is the most challenging task to be explored in modern cosmology. In order to solve this problem, exact formulation of relevant mathematical theory for cosmology is inevitable.

It may be worth to make sure of an important point associated with the dynamics of the scalar field, which is apt to be passed over in this context. Because the dimension of scalar field is different from that of the usual position variable in ordinary mechanics, one is liable to make a mistake when unfolding quantum field theory. In this paper, we point out a problem that appears in the development of quantum theory with inflation universe model. Many researchers define the quantum field momentum as (Boyanovsky et al. 1994; de M Carvalho et al. 2004; Lopes et al. 2009; Gao et al. 1996)

$$\hat{\pi}_{\mathrm{k}} = -i\frac{\partial}{\partial\phi_{\mathrm{k}}}. \qquad (1)$$

However, we will show in this work that this definition raises a dimensional problem in expanding cosmological theory. This problem is an obstacle in unfolding cosmology and must be settled for further progress of the cosmology. The purpose of this research is to suggest some methods to fix this problem.

2. A PROBLEM IN QUANTUM COSMOLOGY

We develop a cosmological theory of the early universe described by a scalar field and check what is the problem of the existing cosmology. Let us consider a simple inflation (scalar field) model in FRW (Friedmann-Robertson-Walker) spacetime with a line element of the form

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \qquad (2)$$

If we denote the time-dependent potential for k-th mode scalar field as $V_{\mathrm{k}}(\phi_{\mathrm{k}}, t)$, the energy density of the universe is given by

$$\rho_{\mathrm{k}} = T_{\mathrm{k}} + V_{\mathrm{k}}(\phi_{\mathrm{k}}, t), \qquad (3)$$

where $T_{\mathrm{k}} = \dot{\phi}_{\mathrm{k}}^2/2$ and ϕ_{k} is the k-th mode scalar field. The equation of motion for the scalar field is represented as (Suresh, 2004; Boyanovsky et al. 1994; Kim, 2012; Kim et al. 2004; Weinberg, 2000; Suen et al. 1988)

$$\ddot{\phi}_{\mathrm{k}} + 3\frac{\dot{a}}{a}\dot{\phi}_{\mathrm{k}} + V_{\mathrm{k}}'(\phi_{\mathrm{k}}, t) = 0, \qquad (4)$$

where $V_{\mathrm{k}}'(\phi_{\mathrm{k}}, t) = \partial V_{\mathrm{k}}(\phi_{\mathrm{k}}, t)/\partial\phi_{\mathrm{k}}$ Notice that this equation involves a term relevant to nonconservative force ($\dot{\phi}_{\mathrm{k}}$ -term) acting on the evolution of the scalar field.

Now we will show how to derive generalized Langrangian density that corresponds to the equation of motion given in Eq. (4). Regarding that Eq. (4) involves a dissipation term, we introduce a generalized force as

$$Q_{\mathrm{k}} = -V_{\mathrm{k}}'(\phi_{\mathrm{k}}, t) + \tilde{Q}_{\mathrm{k}}, \qquad (5)$$

where $\tilde{Q}_{\mathrm{k}}$ is a nonconservative force. We can confirm from Eq. (4) that nonconservative force is given by $\tilde{Q}_{\mathrm{k}} = -3\dot{a}\dot{\phi}_{\mathrm{k}}/a$.
Then, the generalized potential U_{k} is obtained from (Goldstein, 1980)

$$\frac{d}{dt}\frac{\partial U_k}{\partial \dot{\phi}_k}-\frac{\partial U_k}{\partial \phi_k}=Q_k. \quad (6)$$

A rigorous evaluation of this equation yields

$$U_k=\frac{1}{2}\dot{\phi}_k^2[1-a^3(t)]+a^3(t)V_k(\phi_k,t). \quad (7)$$

Considering that the generalized Langrangian density is defined as

$$\mathcal{L}_k=T_k-U_k, \quad (8)$$

we easily get the corresponding generalized Langrangian density in the form

$$\mathcal{L}_k(\phi_k,\dot{\phi}_k,t)=a^3(t)\left[\frac{1}{2}\dot{\phi}_k^2-V_k(\phi_k,t)\right]. \quad (9)$$

The momentum of the field is defined in terms of the Langrangian density, such that $\pi_k=\partial\mathcal{L}_k/\partial\dot{\phi}_k$.

By taking advantage of this Langrangian density, we can easily derive the classical Hamiltonian density to be

$$\mathcal{H}_k(\phi_k,\pi_k,t)=\frac{\pi_k^2}{2a^3(t)}+a^3(t)V_k(\phi_k,t), \quad (10)$$

where

$$\pi_k=a^3\dot{\phi}_k. \quad (11)$$

One frequently uses this Hamiltonian in order for study of the early state of the universe (Kim et al. 2004; Suresh, 2004; Boyanovsky et al. 1994; Kim, 2012).

The usual formula of Hamilton's equations with the Hamiltonian represented in Eq. (10) are given by

$$\dot{\phi}_k=\frac{\partial\mathcal{H}_k(\phi_k,\pi_k,t)}{\partial\pi_k}, \quad (12)$$

$$\dot{\pi}_k=-\frac{\partial\mathcal{H}_k(\phi_k,\pi_k,t)}{\partial\phi_k}. \quad (13)$$

We can see that the equation of motion, Eq. (4), is immediately derived using the above equations. Thus, there is no problem in classical dynamics developed up to now.

The quantum Hamiltonian corresponding to Eq. (10) is obtained by replacing ϕ_k and π_k by their corresponding operators $\hat{\phi}_k$ and $\hat{\pi}_k$ from the classical Hamiltonian:

$$\hat{\mathcal{H}}_k(\hat{\phi}_k,\hat{\pi}_k,t)=\frac{\hat{\pi}_k^2}{2a^3(t)}+a^3(t)V_k(\hat{\phi}_k,t), \quad (14)$$

which agrees with our previous researches (Choi, 2007; Choi, 2011; Choi, 2001; Choi, 2009; Choi, 2013). The total Hamiltonian is $\hat{\mathcal{H}}=\sum_k\hat{\mathcal{H}}_k(\hat{\phi}_k,\hat{\pi}_k,t)$.

The difficulty for this procedure of quantum formulation associated with Eq. (14) is that we are impossible to define the quantum counter field momentum in a usual way in quantum mechanics (under $\hbar=c=1$), i.e.,

$$\hat{\pi}_k\neq -i\frac{\partial}{\partial\phi_k}. \quad (15)$$

This difficulty is taken place from the disagreement of dimension for both sides of Eq. (15). Notice that the dimension of right hand side of the above equation is (Energy)$^{-1}$ while that of Eq. (11) is (Energy)^2. For this reason, many authors (Suresh, 2004; Kim, 2012; Kim et al. 2004; Kim, 2007; Bak et al. 1998) develop their quantum cosmology without mentioning the explicit form of quantum field momentum which is the quantum counterpart formula of the classical field momentum given in Eq. (11). According to this, the definition of field momentum given in lots of reports, including the first equation on page 2771 of Ref. (Boyanovsky et al.

1994), Eq. (14) of Ref. (de M Carvalho et al. 2004), Eq. (14) of Ref. (Lopes et al. 2009), the formula after Eq. (3.8) of Ref. (Gao et al. 1996), and so on, is inadequate. This may be a nonnegligible problem in the existing quantum field theory.

3. IMPROVED FORMULATION OF QUANTUM FIELD THEORY

Now we will search some methods for settling the problem raised in the previous section. A useful commutation relation that enables us to fix the problem is found in Eq. (3.11) of Ref. (Guth et al. 1985), Eq. (3.6) of Ref. (Gao et al. 1996), and Eq. (1.3) of Ref. (Guven et al. 1989). If we rewrite it according to our notation, we get

$$[\hat{\phi}_{k}(\mathrm{r},t),\hat{\pi}_{k'}(\mathrm{r}',t)] = i\delta_{k,k'}\delta^{d}(\mathrm{R}) = \frac{i}{|\mathrm{R}|^{d}}\delta_{k,k'}\delta^{d}(\hat{\mathrm{R}}), \qquad (16)$$

where d represents the order of dimension, R = r-r′, and R̂ is the unit vector in the direction of R, i.e., R̂ = R/|R|. In case of $d=3$, we see that

$$|\mathrm{R}|^{d(=3)} = |x-x'|\cdot|y-y'|\cdot|z-z'| = \mathcal{V}, \qquad (17)$$

where $\mathcal{V}$ is a quantity that has dimension of volume. This may imply that we should regard $\mathcal{V}$ when we define $\hat{\pi}_{k}$. We can take $\mathcal{V}$ in a way that $\mathcal{V}^{-1/3}$ become the energy scale of the universe at a certain boundary time in the early universe (Choi, 2011). As far as we know, most of authors did not consider the volume $\mathcal{V}$ when constructing quantum Hamiltonian density for scalar field associated with the early universe.

According to this, the quantum operator $\hat{\pi}_{k}$ can be redefined instead of Eq. (1) as

$$\hat{\pi}_{k} = -\frac{i}{\mathcal{V}}\frac{\partial}{\partial\phi_{k}}, \qquad (18)$$

which has been regarded $\mathcal{V}$. Then, the commutation relation becomes

$$[\hat{\phi}_{k},\hat{\pi}_{k'}] = \frac{i}{\mathcal{V}}\delta_{k,k'}. \qquad (19)$$

As you can see, this is consistent with Eq. (16) with $d=3$ Indeed, if we use Eq. (18) as the quantum field momentum instead of Eq. (1), the quantum Hamiltonian given in Eq. (14) is free from the problem of dimension.

Another way to remedy the difficulty is to introduce a somewhat different canonical field momentum instead of Eq. (11), such that

$$\Pi_{k} = a^{3}\mathcal{V}\dot{\phi}_{k}. \qquad (20)$$

In this case, the Hamilton's equations are of the form

$$\dot{\phi}_{k} = \mathcal{V}\frac{\partial\mathcal{H}_{k}(\phi_{k},\Pi_{k},t)}{\partial\Pi_{k}}, \qquad (21)$$

$$\dot{\Pi}_{k} = -\mathcal{V}\frac{\partial\mathcal{H}_{k}(\phi_{k},\Pi_{k},t)}{\partial\phi_{k}}. \qquad (22)$$

The corresponding classical Hamiltonian is given by

$$\mathcal{H}_{k}(\phi_{k},\Pi_{k},t) = \frac{\Pi_{k}^{2}}{2a^{3}(t)\mathcal{V}^{2}} + a^{3}(t)V_{k}(\phi_{k},t). \qquad (23)$$

Thus, by replacing classical variables with quantum operators, the quantum Hamiltonian is expressed in the form

$$\hat{\mathcal{H}}_{k}(\hat{\phi}_{k},\hat{\Pi}_{k},t) = \frac{\hat{\Pi}_{k}^{2}}{2a^{3}(t)\mathcal{V}^{2}} + a^{3}(t)V_{k}(\hat{\phi}_{k},t), \qquad (24)$$

where the quantum field momentum operator is defined as

$$\hat{\Pi}_{\mathrm{k}} = -i\frac{\partial}{\partial \phi_{\mathrm{k}}}. \quad (25)$$

Then it satisfies $[\hat{\phi}_{\mathrm{k}}, \hat{\Pi}_{\mathrm{k'}}] = i\delta_{\mathrm{k,k'}}$. We can easily confirm that the quantum Hamiltonian, Eq. (24) with Eq. (25), also does not raise dimensional problem, leading to exact approach for quantum field theory.

The two new formulations of quantum field theory we developed here are mathematically consistent. The only difference between them is the way for representing the theory with slightly different definition of the quantum field momentum. Apparently, these recipes provide proper description of quantum field theory associated with the scalar field.

4. CONCLUSION

From fundamental dynamics of cosmology with the Langrangian, we can show that the Hamiltonian associated with the inflation universe is given by Eq. (10). But we have found that the previous researches on quantum field theory accompany an illusive problem. That is some mismatch of fundamental dimensions, appearing when we unfold quantum field theory on the basis of existing definition of quantum field momentum.

We showed that quantum field momentum should be determined regarding the volume in a way that it should be expressed as Eq. (18). Then, the dimensional problem does not take place in the description of the cosmology. Another method to settle the problem is to choose the quantum field momentum in a way given in Eq. (25) together with the Hamiltonian expressed as Eq. (24).

Notice that these prescriptions for quantum field theory are deeply related with the volume $\mathcal{V}$. We can regard that $\mathcal{V}$ is a characteristic volume that can be chosen in a way that $\mathcal{V}^{-1/3}$ become the energy scale at a particular boundary time that we consider in the development of cosmology (Choi, 2011).

From scrutinized survey of previous reports in this field, we can confirm that most of the researchers in cosmology are unaware of the problem pointed out here. However, in fact, our early developments (Choi, 2007; Choi, 2011; Choi, 2001; Choi, 2009; Choi, 2013) of cosmology are performed regarding this so that they do not raise dimensional problem when describing quantum cosmology and, as a consequence, we showed the possibility that the cosmological constant problem can be resolved without resorting to exotic theory or introducing special matter as the identity of the dark energy. The exact theoretical formulation of cosmology with appropriate description of the scalar field dynamics may be crucial for successful development of cosmology.

SUMMARY OF RESEARCH

1. It is shown how to describe quantum cosmology of the early universe through correct definition of quantum field momentum.
2. The new description of the dynamics of scalar fields in this work is adequate as well as precise since it does not raise dimensional problem.

FUTURE ISSUES

Very recently (March 17, 2014), Harvard–Smithsonian Center for Astrophysics has announced that the evidence of inflation of the Universe at the early cosmic epoch is verified from the data of BICEP2 which is an astronomical instrument located at Earth's south pole. Moreover, it is turned out from WiggleZ Dark Energy Survey (Blake C et al., 2011a and b) and SNLS3 (Sullivan M et al., 2011) that the identity of the mysterious dark energy is the cosmological constant. According to these observations, one can carry out the research for the evolution of the vacuum energy density on the basis of inflation model of cosmology with correct description of scalar field theory which is represented here.

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