



Nonlinear time series analysis of rainfall over central Indian region using CMIP5 based climate model

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General Note



Article is recommended to print as color version in recycled paper. *Save Trees, Save Climate.*

ABSTRACT

In present study, nonlinear time series analysis is employed to examine the predictability and existence of chaos in the Indian Summer Monsoon Rainfall (ISMR) over central Indian region using Intergovernmental Panel on the Climate Change (IPCC) Coupled Model Inter-comparison Project 5 (CMIP5) models and observations from the India Meteorological Department (1901-2006) for central Indian region. The climate projections under different Representative Concentration Pathways (RCPs) namely, RCP 2.6, 4.5, and 8.5 are used. The model data of 146 years (1860-2005) of historical simulation and 295 years (2006-2300) of future projections

from the CMIP5 models under RCP 2.6, 4.5, and 8.5 scenarios are taken. The techniques of correlation dimension and largest Lyapunov exponents are used to quantify chaos and estimate predictability of the ISMR. The Grassberger-Procaccia algorithm is employed in calculating the correlation dimension which gives minimum number of variables required to model the dynamics of system. The average divergence rate of nearby orbits is given by the largest Lyapunov exponent. Presence of chaos is evident by the fractal value of correlation dimension and positive largest Lyapunov exponent for all the data sets. Predictability is calculated for each scenario using largest Lyapunov exponent.

Keywords: Chaos, Correlation dimension, CMIP models, Lyapunov exponent, RCP scenarios.

1. INTRODUCTION

Scientists and policy makers must consider the budding impact of global warming and climate change, as it can have huge impact on our planet's people and ecosystem (Biswas Roy et al. 2015; Rishma et al. 2015). So it is important to have an idea about the concentration of carbon dioxide and other green house gases (GHGs). The concentration of these gases depends on natural causes and anthropogenic activities. CO₂ and other GHGs traps heat radiating from earth towards the space, thereby affecting the radiative forcing (Jyh-Woei, Lin, 2015). Change in radiative forcing is a key factor for climate change. Taking this change in radiative forcing into account climate projections for fifth assessment report of Intergovernmental Panel on Climate Change (IPCC) are given using newly developed Representative Concentration Pathways under fifth phase of Coupled Model Inter-comparison Project (CMIP5). The representative concentration pathways (RCPs) are green house gas concentration trajectories adopted by IPCC. The RCPs, RCP2.6, RCP4.5 and RCP8.5, are named according to possible total radiative forcing values in the year 2100, for example RCP8.5 means the value of radiative forcing will be 8.5 W/m² in 2100.

In present study, methods of nonlinear time series analysis are employed to examine predictability and chaos using rainfall data from CMIP5 based climate models namely, BCC-CSM1.1, MPI-ESM-LR, HadGEM2-ES and IPSL-CM5A-LR and rainfall data from IMD (India Meteorological Department) for central Indian region, geographically situated at 15°N - 30°N latitude and 70°E - 90°E longitude. The model data includes historical simulation (1860-2005) and future projections RCP 2.6, 4.5, and 8.5 scenarios (2006-2300).

The systems, which have sensitive dependence on initial conditions, are called chaotic systems. The Earth's atmosphere is an example of a chaotic system; irreducible errors in and incompleteness of observations of the initial conditions as well as the imperfection of the models prohibit accurate forecasts of the day-to-day sequence of weather beyond a few days. The atmosphere and ocean are mathematically modeled as nonlinear dynamical systems, and even an infinitesimally small uncertainty in the initial conditions will grow exponentially to make the predictions lose practical value after a finite amount of time.

Modeling of deterministic dynamical system is based on the concept of phase space, the collection of all the possible states. Dynamical system describes how one state develops into another state over the course of time. Since the dynamics of chaotic time series like rainfall is unknown and information about all the variables is not available. Thus the phase space reconstruction provides a simplified, multi-dimensional representation of a single dimension time series. The physics behind such an assumption is that a single record from a dynamical system is the outcome of all interacting variables and the single variable carries the information about entire system and the evolution of trajectories in the reconstruction space, in a sense, mimics the behavior of actual trajectories in full state space. Packard et al., 1980 and Takens, 1981 described the time delay coordinate method to approximate the state space from a scalar time series. According to this method, for a scalar time series X_i where $i = 1, 2, 3, \dots, N$, can be fully embedded in m dimensional space represented by a vector, $Y_j = (X_j, X_{j+\tau}, X_{j+2\tau}, \dots, X_{j+(m-1)\tau})$, where τ is suitably chosen delay or lag time and m is the embedding dimension.

2. METHODS

2.1. Correlation Dimension

Correlation dimension is one of the most efficient methods to determine the existence of chaos and fractal dimension, it can be estimated by Grassberger-Procaccia algorithm. Correlation dimension gives minimum number of variable needed to describe the state of a system. Let us consider a trajectory, which is a collection of N data points, assuming that X_i , $i = 1, 2, \dots, N$ be the single

variable time series of observed data, where N is the total number of observation. The procedure usually begins with reconstructing the data in m dimensional phase space using the method of time delay from the observed time series (Packard et al., 1980; Takens, 1981). The reconstructed vectors are $Y_j = (X_j, X_{j+\tau}, X_{j+2\tau}, \dots, X_{j+(m-1)\tau})$ where τ is suitably chosen time delay and m is the embedding dimension. The correlation integral is defined as the ratio of number of pairs having distance less than an arbitrary r to the total number of pairs (Grassberger and Procaccia, 1983).

Mathematically,

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{i,j=1}^{\infty} \theta(r - |X_i - X_j|) \quad (2.1)$$

Where θ denotes the Heaviside step function, for $u = r - |X_i - X_j|$, $\theta(u) = 0$, if $u < 0$ and $\theta(u) = 1$, if $u > 0$. For small values of separation distance r the correlation integral $C(r)$ has been found to follow a power law such that $C(r) \approx r^D$, where D is known as correlation dimension. The slope of $\log(C(r))$ vs. $\log(r)$ for a particular value of embedding dimension m gives the correlation dimension D .

The value of D is calculated at different embedding m , initially with increasing the value of embedding dimension m for a chaotic time series the value of D also increases and then it becomes independent of m and attains a saturation value (as shown in figure 1(b)), this saturated value of D is referred as correlation dimension (D_s) and the value of embedding dimension m_s after which saturation occurs gives the upper bound of number of variables needed to describe the system.

2.2. Largest Lyapunov Exponent

Chaotic systems have limited predictability which is a consequence of sensitive dependence on initial conditions; small uncertainty in initial condition can lead to wrong forecast after finite steps. Lyapunov exponents describe the rate of divergence or convergence of two nearby trajectories in phase space i.e., it describes the rate at which predictability is lost (Dwivedi et al., 2007). There is one exponent for each dimension; a positive Lyapunov exponent is a strong indicator of chaos. In a given time series we shall label the series $x(t_0), x(t_1), x(t_2), \dots$ as x_0, x_1, x_2, \dots . t_0 and t_n are zeroth and n^{th} time step respectively, $t_0 - t_n = n\tau$. Selecting some value of x say x_i , then the distance between x_i and another point x_j which is close to x_i be d_0 i.e., $d_0 = |x_i - x_j|$ then the sequence of differences $d_1 = |x_{i+1} - x_{j+1}|$, $d_2 = |x_{i+2} - x_{j+2}|$, $d_n = |x_{i+n} - x_{j+n}|$ is assumed to increase exponentially, therefore one can write $d_n = d_0 e^{\lambda n}$ after taking logarithm $\lambda = \frac{1}{n} \ln\left(\frac{d_n}{d_0}\right)$. This λ is called Lyapunov exponent. Since the rate of separation is different for various orientations of initial separation vector, total number of Lyapunov exponent is equal to the number of dimension of the phase space defined, i.e. a spectrum of Lyapunov exponent will be available. In many practical cases it is sufficient to calculate the largest Lyapunov exponent. The largest Lyapunov exponent is defined as

$$\lambda = \lim_{n \rightarrow \infty} \frac{1}{n\tau} \sum_{k=1}^n \ln \left| \frac{d_k}{d_{k-1}} \right| \quad (2.2)$$

Where d_0 and d_k represents, respectively, the distance between nearby trajectories in the phase space at the initial time and at time $k\tau$ (Wolf et al., 1985).

In this paper we have used the method suggested by Parlitz (1998). In this method, the exponential divergence of neighboring orbits is estimated on a logarithmic time scale via prediction error. A true state or reference state on the reconstructed attractor at time $t = nk\tau$ is given by y^n and the predicted or neighboring state by y^{nn+k} . Prediction error $p(k)$ for k step in advance is given by

$$p(k) = \frac{1}{nk\tau} \sum_{n=1}^N \log \left(\frac{\|y^{n+k} - y^{nn+k}\|}{\|y^n - y^{nn}\|} \right) \quad (2.3)$$

The slope of linear segment of prediction error $p(k)$ vs. number of time steps k gives the measure of Largest Lyapunov exponent.

3. RESULTS

Nonlinear time series analysis is performed in the context of climate change for the months JJAS (June-September) on central Indian region by the methods discussed in section 2 using 295 years (2006-2300) of rainfall data of RCP 2.6, 4.5 and 8.5 scenario, 146 years (1860-2005) of rainfall data from historical simulation from different CMIP5 based climate models and 113 (1901-2013) years IMD data.

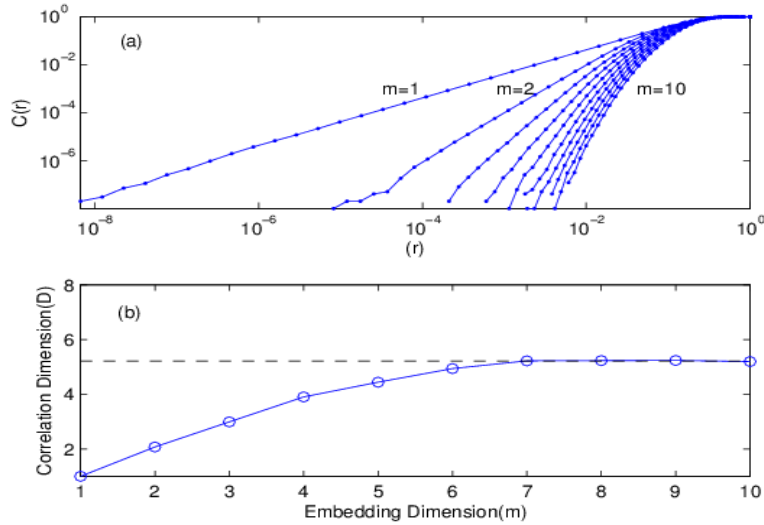


Figure 1 (a) log $C(r)$ as a function of log r for embedding dimension $m = 1$ to 10 (from left to right) for IMD rainfall data, (b) correlation dimension D as a function of embedding dimension m . Horizontal dashed line shows $D_s = 5.23$.

Correlation integral $C(r)$ is calculated according to Grassberger-Procaccia algorithm using equation (2.1) for embedding dimension 1 to 10. The plot of $\log(C(r))$ vs. $\log(r)$ for embedding dimension m ranging from 1 to 10 is shown in fig 1(a) for IMD data. For each embedding dimension m , slope of $\log(C(r))$ vs. $\log(r)$ gives the corresponding correlation dimension. Fig 1(b) shows that correlation dimension is plotted as function of embedding dimension. The correlation dimension increases with increases in embedding dimension up to $m = 6$ and then it attains a saturation value after $m = 7$. The saturation correlation dimension, $D_s = 5.23$ represents chaotic nature of IMD data. The nearest whole number to the saturation correlation dimension 5.23 is 6, thus minimum of 6 variables and maximum 7 variables are required to describe the state of the system.

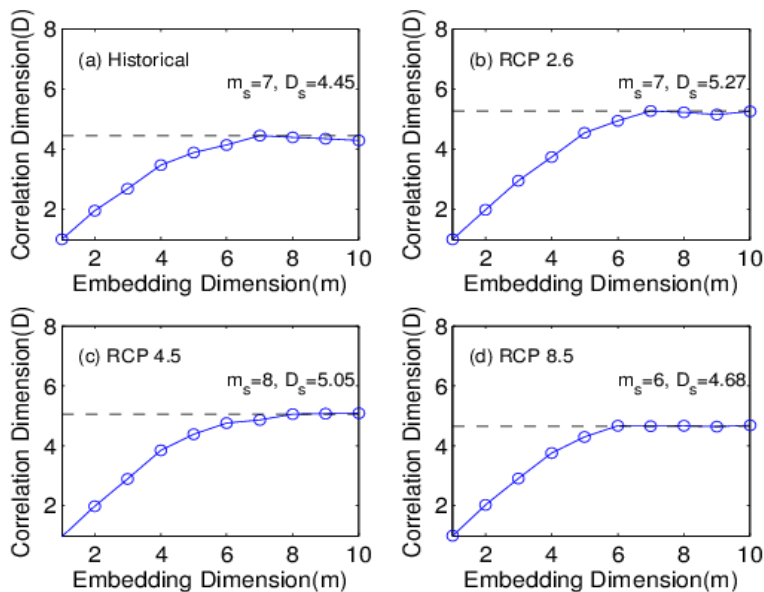


Figure 2 Correlation Dimension D as a function of embedding dimension m for different scenarios of BCC-CSM1.1

Similarly, the correlation dimension for historical simulation and RCP scenarios for different CMIP5 based climate models, namely, BCC-CSM1.1, MPI-ESM-LR, HadGEM2-ES and IPSL-CM5A-LR are calculated and their values are given in table 1. Figure 2 shows the variation of correlation dimension D as function of embedding of m for different scenarios of BCC-CSM1.1 model. Figures for other models are not shown in this paper. It is found that the non-integer value of saturation correlation dimension for all four models ranges between 4 and 6 ($4 < D_s < 6$) and the value of saturation embedding dimension, m_s is either 6 or 7 except RCP4.5 scenario of BCC-CSM1.1 having saturation embedding dimension, $m_s = 8$. Thus the non-integer (fractal) values of saturation correlation dimension (D_s) indicate the presence of chaos in every scenario of different models.

Table 1 Saturation correlation Dimension D_s and saturation embedding dimension m_s for all four scenarios of different CMIP5 bases climate models

Scenarios	Saturation correlation dimension(D_s) and saturation embedding dimension(m_s)							
	BCC-CSM1.1		MPI-ESM-LR		HadGEM2-ES		IPSL-CM5A-LR	
	D_s	m_s	D_s	m_s	D_s	m_s	D_s	m_s
Historical	4.45	7	4.38	7	4.42	7	4.12	6
RCP2.6	5.27	7	4.67	7	4.43	7	4.32	7
RCP4.5	5.05	8	4.71	7	4.38	7	4.38	6
RCP8.5	4.68	6	4.59	7	4.52	7	4.64	7

Largest Lyapunov exponent is estimated as described above in section 2.2 using equation (2.3). The slope of the linear part of curve (figure 3) plotted between prediction error and number of time steps k gives the largest Lyapunov exponent.

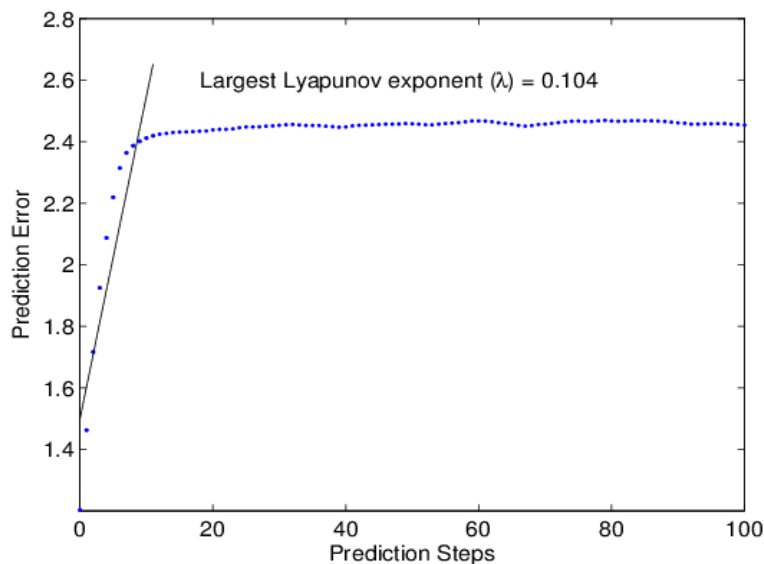


Figure 3 Prediction error $p(k)$ for IMD rainfall data as a function of prediction step k . Slope of the straight (solid) line give the largest positive Lyapunov exponent

A positive slope of around 0.104 for IMD data confirms the exponential divergence of trajectories and hence the chaotic nature of the rainfall over central of India. The inverse of the Lyapunov exponent defines the predictability of the system, which is around 9 days. Similarly we have estimated the maximum Lyapunov exponent and predictability for all four scenarios of above mentioned models and the results are shown in table 2 and 3.

Table 2 Values of Largest Lyapunov Exponent for all four scenarios of different CMIP5 based climate models

Scenarios	Largest Lyapunov exponents for different CMIP5 models			
	BCC-CSM1.1	MPI-ESM-LR	HadGEM2-ES	IPSL-CM5A-LR
Historical	0.140	0.104	0.141	0.099
RCP2.6	0.053	0.054	0.057	0.053
RCP4.5	0.060	0.046	0.058	0.061
RCP8.5	0.055	0.056	0.053	0.058

Table 3 Values of Predictability in days (i.e. inverse of largest Lyapunov exponent) for all four scenarios of different CMIP5 based climate models

Scenarios	Predictability in days for different CMIP5 models			
	BCC-CSM1.1	MPI-ESM-LR	HadGEM2-ES	IPSL-CM5A-LR
Historical	7	9	7	10
RCP2.6	18	18	17	18
RCP4.5	16	21	17	16
RCP8.5	18	17	18	17

4. CONCLUSIONS

In this paper, an attempt has been made to investigate the predictability and existence of chaotic behavior in rainfall data. Overall study has been done in the light of expected climate change in response to increase in emission of carbon dioxide and other green house gases. We focused our study over central Indian region (15°N - 30°N latitude and 70°E - 90°E longitude) for the months JJAS (June-September) using 113 years (1901-2013) IMD (India Meteorological Department) rainfall data, 295 years (2006-2300) RCP scenarios, 146 years historical scenario data of BCC-CSM1.1, MPI-ESM-LR, HadGEM2-ES and IPSL-CM5A-LR models. The analysis is based on two main chaos quantifiers namely correlation dimension and Lyapunov exponent. The results obtained from correlation dimension method i.e. fractal value of correlation dimension, confirms the presence of deterministic chaos. The positive value of largest Lyapunov exponent shows the finite predictability for all data sets used in the analysis. Predictability varies from 7 to 10 days (table 3) for historical scenarios (1860-2005) showing good agreement with IMD data (1901-2006) having predictability of 9 days. The values of predictability for RCP 2.6, 4.5 and 8.5 scenarios for different CMIP5 models is ranging between 16 to 18 days, except for RCP4.5 scenario of MPI-ESM-LR model having predictability of 21 days. The difference of predictability between historical simulation and future projections (RCP scenarios) indicates that rainfall events in future will be more predictable.

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