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DISCOVERY

Description of motion of system of particles in different resisting media

Alalibo T Ngiangia[1], Eqhuanoye S Ikata[2], Peter O Nwabuzor[1]

[1]Department of Physics, University of Port Harcourt, P M B 5323, Choba, Port Harcourt, Nigeria
[2]Department of Physics, Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt, Nigeria

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ABSTRACT
A numerical study of the description of motion of system of particles in different resisting media was carried out. The paper transforms the time dependent Newton's equation of motion in a resisting medium into a space dependent form which necessitated the use of classical Runge-Kutta method of order four to determine the solution of the model. The work which focused on resisting medium proportional to exponent of velocity, showed that increase in the mass of a particle in an air resisting medium results in a corresponding increase in the velocity of the particle while an increase in the density of the resisting medium led to a decrease in the velocity of the particle which is not different in the relativistic medium. An algorithm on the application of Runge-Kutta method of order four is also proposed to tackle models of this nature.

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1. INTRODUCTION

Generally, an object or particle thrown upward or allowed to fall from a height is acted upon by its weight and a host of other forces as well. Essential classes of forces for our consideration are those that tend to oppose the motion of an object or particles. These classes of forces arise because of motion in some media especially fluids referred to as resisting, damping or dissipative forces and the corresponding media is said to be a resisting media. For dynamic systems, a physically valid formula expressing the resistive force as a function of the velocity is yet to be ascertained [1] or the resistive force can have complicated velocity dependence [2]. However, experiments show that for low velocities, the resisting force is in magnitude proportional to the velocity. But this simple relationship breaks down before we reach those velocities which are of interest in ballistics. For low ballistics, the resistive force varies proportionally to the square of the velocity, but the law again breaks down when the velocity of the particle or object approaches the velocity of sound. The law of dependence is then complicated and can only be represented by experimentally obtained values. Therefore, a simple relationship for particle trajectories in a resistant medium is still a challenge till date. This difficulty, posed to researchers to propose a law or working relationship of particles in a resistant medium have not in any way slow down the investigation of particles motions in a resistant medium using different approximations and assumptions [2]. A study on the effect of Hartmann number on the part of a particle in a resistant medium was carried out by [3] wherein the conditions of the resistive force proportional to both the velocity and square of the velocity were examined and far reaching conclusions made. [4], studied the motion of a particle in a resisting medium using fractional approach and proved that, there exist a relationship between the coordinate and the fractional component in a given system through a physical parameter. [5], investigated the motion of a particle through a resisting medium of variable density. In the study, he applied a resistance proportional to the product of instantaneous velocity and air density and opined that the density diminishes exponentially with the altitude. The vertical projection in a resisting medium: reflections on observations of Mersenne, was critically examined by [6]. The study validates empirical observations of Marin Mersenne on timing of vertically-launched projectiles for a general mathematical model of resistance. In a study, [7], proposed a fractional differential equation describing the behaviour of two dimensional projectile motions in a resisting medium and opined that the trajectories of the projectile in the fractional approach are always less than that of the classical case. Studies of [8, 10], examined the effect of magnetic field on projectile motion in a resisting medium and reported that, increase in magnetic field result in a decrease in the velocity of the particle. [11], included electric field to the study of [8] and deduced similar findings. [12], in the study of horizontal and vertical projectile motion in a resistant medium subject to varying path angles and speed, showed that increase in path angle decrease the projectile motion. Our aim is to compare the behaviour of system of particles in an air resisting medium considering the non relativistic case and the relativistic case with a view to proposing a model to describe the different velocity dependent resisting function.

Classical Runge-Kutta Method of Fourth Order

The Runge-Kutta formula involves a weighted average of values of $f(y,\vec{v})$ taken at different points in the interval $\vec{y}_n \le \vec{y} \le \vec{y}_{n+1}$. It is given by [13]

$$\vec{v}_{n+1} = \vec{v}_n + \frac{h}{6}\left(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}\right) \qquad (1)$$

where

$$k_{n1} = f\left(\vec{y}_{n,}\vec{v}_n\right)$$

$$k_{n2} = f\left(\vec{y}_{n,} + \frac{h}{2}, \vec{v}_n + \frac{h}{2}k_{n1}\right)$$

$$k_{n3} = f\left(\vec{y}_{n,} + \frac{h}{2}, \vec{v}_n + \frac{h}{2}k_{n2}\right)$$

$$k_{n4} = f\left(\vec{y}_{n,} + h, \vec{v}_n + hk_{n3}\right)$$

2. FORMALISM

A system of particles of mass (m) moving with an initial velocity 10m/s in a resisting medium whose steady resistance to the motion of the particle acts instantaneously in the opposite direction to that of the particles motion. The mass of the particle experiences, the weight downward and the resisting force upward. The study assumed that;

The resistance medium is air.

The acceleration due to gravity is constant over the range of motion and directed downwards.

The rotation of the earth does not affect the motion of the particle.

The resistance or damping to the motion is a continuous function of the velocity satisfying natural physical requirements.

The upward buoyant force is neglected

With these assumptions, the Newton's law of motion in the presence of a resistance proportional to the velocity is given as

$$\frac{md^2\vec{r}}{dt^2} = mg - \frac{1}{2}\beta_1 \rho A \vec{v} \qquad (2)$$

where m is mass of particle, $\vec{r}$ is position vector, g is acceleration due to gravity, β_1 is a positive dimensionless proportional constant that specifies the strength of the resisting force, t is time, ρ is density of air, A is cross-sectional area of the object and $\vec{v}$ is velocity vector.

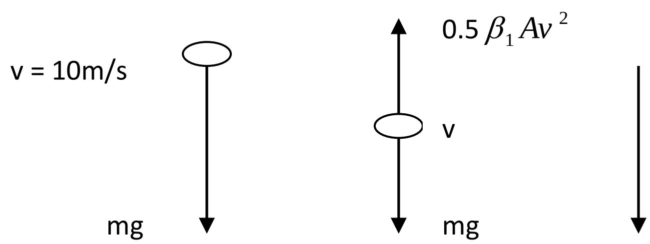

Figure 1 Model and coordinate system of the problem

Equation (2), can be written in terms of vertical space coordinate as

$$\frac{d\vec{v}}{d\vec{y}} = \frac{g}{\vec{v}} - \frac{\beta_1 \rho A}{2m} \qquad (3)$$

with the initial condition $v(0) = 10m/s$

3. METHOD OF SOLUTION

Case 1

Non relativistic

For comparatively low velocities as observed in the case of damped simple harmonic oscillator and parachutist, equation (3) is a model for these descriptions. The solution to equation (3) is difficult to tackle analytically. The classical Runge-Kutta method of order four as stated in equation (1) is applied with a step size (h) = 0.1. Other parameters used are $\beta_1 = 0.5$, m(kg) = 0.145, 0.245, 0.345, 0.445, 0.545. $g = 9.81m/s^2$, $A = 0.0042m^2$ $\rho = 1.2754kg/m^3$ $c = 3x10^8 m/s$

Table 1

m(kg)	v(m/s)
0.145	10.86536900
0.245	10.86813181
0.345	10.87008446
0.445	10.87176352
0.545	10.87273650

Table 2

$\rho\left(kg/m^3\right)$	v(m/s)
1.2754	10.8817754
2.2754	10.88164244
3.2754	10.88150701
4.2754	10.88137564
5.2754	10.88122539

Relativistic case

For the relativistic medium, equation (3) takes the form

$$\frac{d\vec{v}}{d\vec{y}} = \frac{g}{\vec{v}} - \frac{\beta_1 \rho A\sqrt{1-\beta^2}}{2m_0} \qquad (4)$$

where $\beta = \frac{v}{c}$, c is velocity of light, m_0 is rest mass

The result of the relativistic case is not different from the non relativistic case.

Case 2

Non relativistic case

For ballistics which is concerned with projectile fired upward from a cannon or small arms and free flight of bombs as well as flight of rockets, the resistance or damping is proportional to the square of velocity. Equation (3) in this situation, transform into

$$\frac{d\vec{v}}{d\vec{y}} = \frac{g}{\vec{v}} - \frac{\beta_1 \rho A v}{2m} \qquad (5)$$

Using the same parameters and method, the results is tabulated as

Table 3

m(kg)	v(m/s
0.145	10.88024478
0.245	10.88094341
0.345	10.88123411
0.445	10.88139407
0.545	10.88149514

Table 4

$\rho\left(kg/m^3\right)$	v(m/s
1.2754	10.88024478
2.2754	10.87827250
3.2754	10.87757757
4.2754	10.87610770
5.2754	10.87490364

Relativistic case

For the relativistic medium, equation (3) takes the form

$$\frac{d\vec{v}}{d\vec{y}} = \frac{g}{\vec{v}} - \frac{\beta_1 \rho A\vec{v}\sqrt{1-\beta^2}}{2m_0} \qquad (6)$$

The result of the relativistic case is not different from the non relativistic case.

Case 3

Non relativistic case

For sounds and others, where the exponent of the velocity is higher than or equal to three, the model transform to

$$\frac{d\vec{v}}{d\vec{y}} = \frac{g}{\vec{v}} - \frac{\beta_1 \rho A \vec{v}^{\,n-1}}{2m_0} \qquad n \geq 3 \qquad (7)$$

Relativistic case

In the relativistic medium, the model takes the form

$$\frac{d\vec{v}}{d\vec{y}} = \frac{g}{\vec{v}} - \frac{\beta_1 \rho A \vec{v}^{\,n-1}\sqrt{1-\beta^2}}{2m_0} \qquad (8)$$

The solution of equations (7) and (8) can be implemented by the algorithm below

Algorithm

Define $f(\vec{y}, \vec{v})$

Input initial values $\vec{y}_0$ and $\vec{v}_0$

Input step size h and number of steps n

Output $\vec{y}_0$ and $\vec{v}_0$

For j from 1 to n do

$k_1 = f(\vec{y}, \vec{v})$

$k_2 = f(\vec{y} + 0.5 * h, \vec{v} + 0.5 * h * k_1)$

$k_3 = f(\vec{y} + 0.5 * h, \vec{v} + 0.5 * h * k_2)$

$k_4 = f(\vec{y} + h, \vec{v} + h * k_3)$

$\vec{v}_1 = \vec{v}_0 + \left(\frac{h}{6}\right) * (k_1 + 2 * k_2 + 2 * k_3 + k_4)$

$\vec{y} = \vec{y} + h$

Output $\vec{y}$ and $\vec{v}$

End

The algorithm can easily be implemented on FORTRAN 9 and Python programming languages.

4. DISCUSSION

Case 1

The mass of a baseball is chosen as the starting mass and subsequently, increase in the mass corresponds to an increase in the velocity through the resisting medium as shown in table 1. An increase in the density of the resisting medium also led to a decrease in the velocity of the baseball because increase in resisting medium has been enhanced as tabulated in table 2.

Case 2

Table 3, showed a relationship between the mass of a particle and its velocity in a resisting medium where the resistive force is proportional to the square of the velocity. For a typical mass of a baseball as used, its increase result in a corresponding increase in the velocity of the particle. This observation remains the same in the relativistic case. The equation (equation 6) shows that as long as v << c, theoretically, the observation in both the relativistic case and the non relativistic case shall remain the same. Table 4 also showed that varying the density of the medium, decreases the velocity of the particle owing to increase resistance of the medium.

Case 3

The pattern of relationship that exist between mass of the baseball in a resisting medium and its velocity and that of density and velocity in case 1 and case 2 is not different from case 3, however, owing to the exponent n, we proposed a mathematical model and a numerical solution using an algorithm. This will serve as a generalized solution of this nature irrespective of the value of n.

5. CONCLUSION

One of the important consequences of special relativity is that mass increases with velocity. This assertion was buttressed in this work. Also, 0.5 is chosen for the dimensionless empirical quantity for sphere called the drag force coefficient which baseball fit into. From observation and some literatures cited, it is observed that objects under which resistive force is proportional to the square of velocity are generally larger than objects under which resistive force is proportional to the velocity. Generally, we observed that the space dependent form of Newton's equation of motion of particles in a resisting medium describes better the path of a particle and the effect of varying other parameters or factors over a long time. Above all, no significant difference between the results of relativistic case and that of non relativistic case of vertical motion of a particle in a resisting medium was observed in this study.

REFERENCE

1. J. L. Synge, B. A. Griffith, Principles of Mechanics, (Third Edition, McGraw-Hill, Kogakusha LTD, 1970)
2. R. A, Serway, Physics for Scientist and Engineers with Modern Physics, (Second Edition, Saunders College Publishing, New York, 1986)
3. A. T. Ngiangia, M. A. Orukari, International Journal of Applied Mathematical Sciences, 5(1), 51 (2012)
4. J. Rosales, M. Guia, J. M. Ortiz, D. Baleanu, Proceedings of the Romanian Academy-Series A: Mathematics, Physics, Technical Sciences, Information Science, 14(1), 42 (2013)
5. R. H. Bacon, American Journal of Physics, 19(1), 64(1951), (2005)
6. C. W. Groetsch, S. A. Yost, The American Mathematical Society, 121(6), 500 (2014)
7. J. J. Rosales, M. Guia, F. Gomez, F. Aguilar, J. Martinez, Central European Journal of Physics, 12(7), 517 (2014)
8. J. C. Hayen, Journal of Non-Linear Mechanics, 38, 357 (2003)
9. O. J. Fenuga, R. O. Ayeni, The Journal of the Mathematical Association of Nigeria, 33(2B), 389 (2006)
10. A. Ngiangia, N. Wonu, F. Jim-George, Journal of the Nigerian Association of Mathematical Physics, 11, 83 (2007)
11. R. O. Ayeni, O. O. Ayandokun, Journal of the Nigerian Association of Mathematical Physics, 7, 169 (2003)
12. F. Life-George, A. T. Ngiangia, P. O. Eke, M. A. Orukari, Journal of the Nigerian Association of Mathematical Physics, 14, 257 (2009)
13. W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, (Fifth Edition, John Wiley and Sons Inc, New York, 1992)